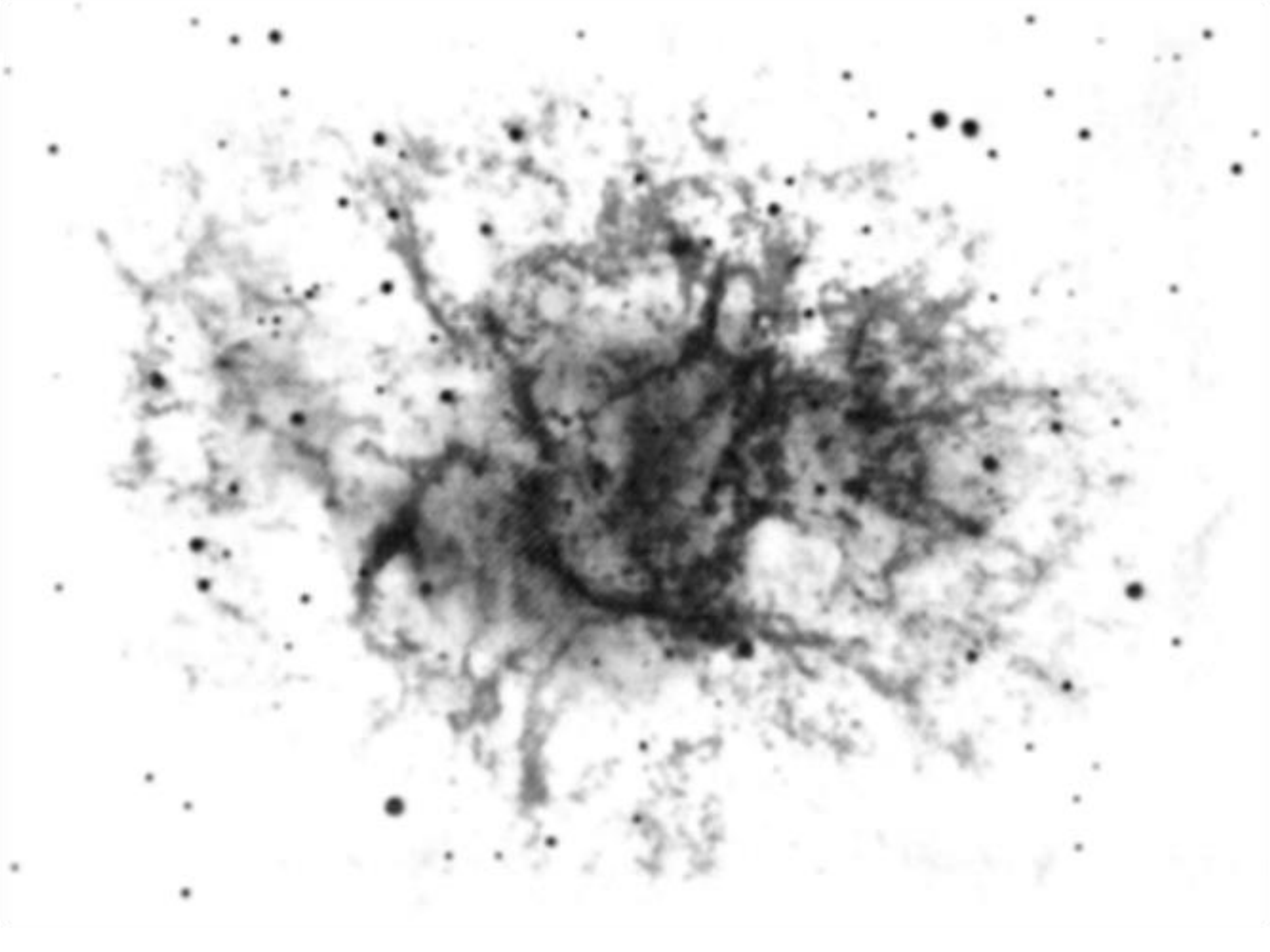
Physics of Astronomy

A-level Physics Option



Alan C Pickwick

Manchester Grammar School

Preface

I have written this booklet for three reasons:

- To encourage more teachers to risk teaching the Physics of Astronomy options.
- To provide students with a set of notes to make them more secure in their studies.
- To allow both students and teachers to enjoy exploring the mysteries of the universe by looking beyond the beautiful photographs into the underlying science.

The text goes far enough to allow a teacher new to the material to use it as the basis for a teaching course but not so far as to prevent a keen student using it as a self-study guide. I would expect that all students be given a complete copy of the text.

I should like to thank the six generations of sixth-formers who used the manuscript version of the booklet and particularly my 1993 A-level group who *really* enjoyed finding all the errors in the first word-processed version! Also recent groups who helped correct and improve this revised edition.

I produced the booklet on a PC using Microsoft Word and illustrated it using CorelDRAW. The headings are set in AvantGarde and the body text in Bookman.

Alan_C_Pickwick@btinternet.com

Manchester Grammar School
(1975-2013 Retired)
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Lenses

The Principal Focus and Focal Length

Rays of light travelling close and parallel to the Principal Axis are refracted by the lens and then pass through the Principal Focus.

On a sunny day you can use a magnifying glass to light a fire. Here the rays from the Sun are almost parallel to each other and so are all focussed at the Principal Focus of the lens, creating great heat.

The Focal Length of a lens is the distance from the centre of the lens to the Principal Focus.

Formation of Images by a Converging Lens

Ray Diagrams

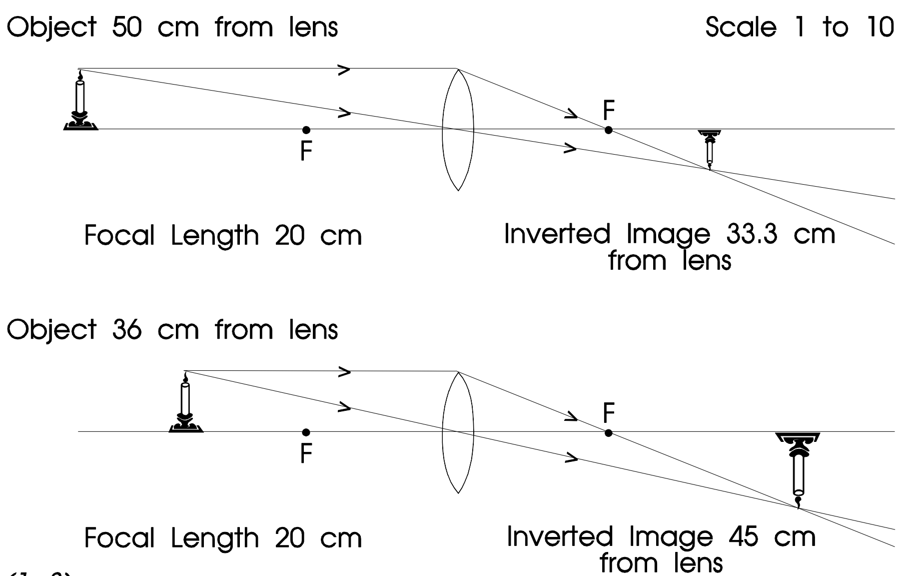
Scale drawings are the easiest way to find out how a lens forms an image. These drawings use a few rays that always behave in simple ways:

- Rays that pass through the centre of the lens carry on in their original straight line
- Rays that approach the lens close and parallel to the optical axis leave the lens heading for the focal point.

The rays are traced from the top of the object and are drawn onwards until they cross. This crossing point shows the location of the image. This is called a real image, because the ray really cross on the far side of the lens.

In the first two examples, the images are inverted and real. You can see a real image on a screen that is placed where the rays cross.

Lenses - Real Images



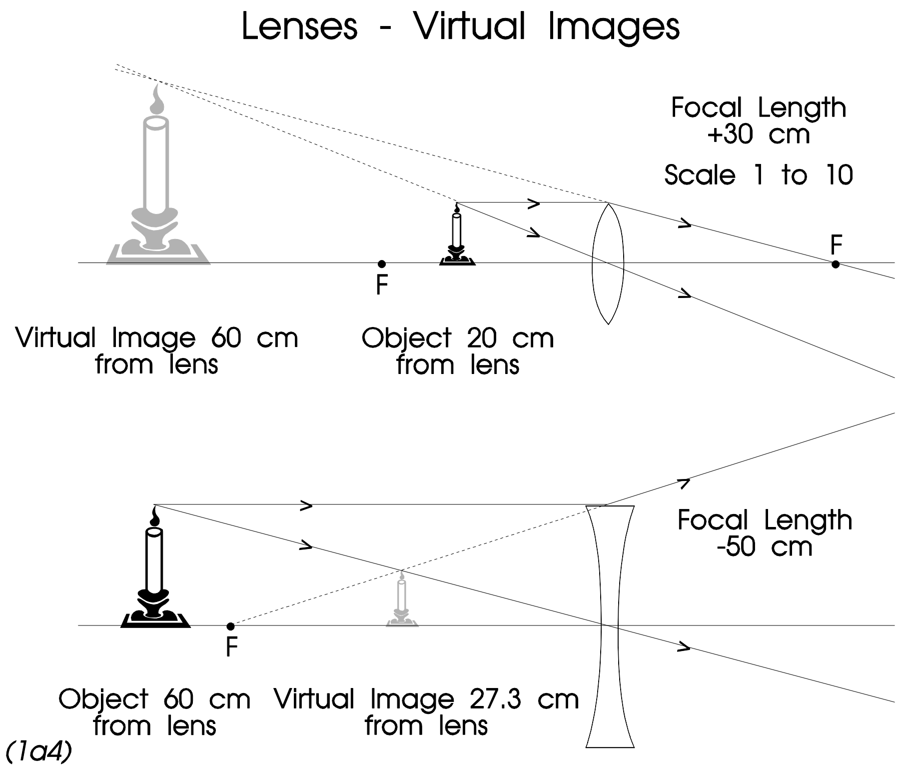
(1a3)

In the first case the image is smaller than the object but in the second case it is larger.

Q - Calculate the magnification of the image by dividing the image height by the object height as measured from the diagrams?
A - First diagram, 0.67. Second diagram, 1.25.

Q - Now confirm these results by dividing the distance of the image from the lens by the distance of the object form the lens. Either method will give the magnification of the image.
A - First diagram, 0.67. Second diagram, 1.25.

These ratios are the magnification of the image compared to the object. You should also be able to see that this follows by comparing the similar triangles on the left and right sides of the diagrams.



In the next two cases the images are virtual. This means that the rays do not cross naturally. To find the image, you have to draw the rays backwards until they do cross. It is important to use a dotted line for this operation, since the rays do not actually travel along these paths.

You cannot project a virtual image onto a screen. However, you can look into the lens with your eye and actually see the image. This is because your eye focuses the rays onto your retina to form a real image. It is interesting to note that your brain thinks that the rays come from the location of the virtual image. This might be quite hard for you to accept, but if you have ever look down a microscope, you have been looking at a virtual image formed about 25 cm in front of your eye. If you wear glasses to correct short sight, then the world you see is all one big virtual image!

In the first drawing the object is between the focal point and the lens. The rays will never cross so they are drawn backwards to find the virtual crossing point. The image is the same way up as the object but is virtual and larger than the object. A magnifying glass works this way.

In the second drawing the lens is of a different type. This diverging lens will never form a real image from a real object because the rays can never cross. The ray that runs parallel to the axis diverges as if it had come from the focal point on the left hand side of the lens. By drawing its path backwards, a virtual crossing point is found. The resulting image is always smaller than the object and is always virtual. Short sight is corrected with a diverging lens.

If you are unsure of the method used to draw the ray diagrams, you should make your own copy of them, marking just the object, lens and focal points. Then apply the standard rules to draw on the rays of light. Check that you get the same image positions and sizes.

The Lens Formula

Mathematical models are useful because they can *predict* results without the need to do experiments. Using the lens formula, you can calculate image positions without having to make scale drawings.

The formula links three distances:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

where u is the object distance,

v is the image distance and
f is the focal length of the lens.

If you prefer, you can remember this formula as:

$$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{\text{focal length}}$$

The formula has some rules for the signs of the distances:

- real object and image distances *are positive*,
- virtual object and image distances *are negative*,

- converging lens focal lengths *are positive*,
- diverging lens focal lengths *are negative*.

If a calculation gives a negative distance, then the image is virtual.

In the table, the image position is predicted for each of the scale drawings of the previous section.

Object Distance / cm	Focal Length / cm	Substitute in formula	Image Distance / cm	Is the image real or virtual? Which side of the lens is the image located?
50	20	$\frac{1}{50}+\frac{1}{v}=\frac{1}{20}$		
36	20			
20	30			
60	-50 Diverging lens	$\frac{1}{60}+\frac{1}{v}=\frac{1}{-50}$		

Answers:
Real Image 33.3 cm from lens on the far side from the object.
Real Image 45.0 cm from lens on the far side from the object.
Virtual Image 60.0 cm from lens and on the same side as the object.
Virtual Image 27.3 cm from lens and on the same side as the object.

Measurement of the focal length of converging lens

The most accurate way to measure the focal length of a lens is to use it to focus a bright object onto a white screen. Record the object and image distances. Collect several more pairs of results by moving the object and then refocusing the image by moving the screen. Use the lens formula to calculate the focal length in each case and then find the average value for higher accuracy. Better still, plot a graph of $1/v$ against $1/u$ and draw the best straight line. The intercepts are both at $1/f$.

Here are some results from such an experiment:

Object Distance /cm	Image Distance /cm	Focal Length/cm
20.0	60.2	
30.0	30.0	
40.0	23.9	
50.0	21.4	
60.0	20.0	
70.0	19.2	
	Average Value:	

Q - Calculate the focal length of the lens that was used.
A - 15.0 cm

Q - Suggest reasons why you would expect a small spread of values in the focal lengths you have calculated?
A - Slight difficulty in focusing the image accurately. Slight difficulty in measuring the position of the lamp, lens and screen.

Extension Material

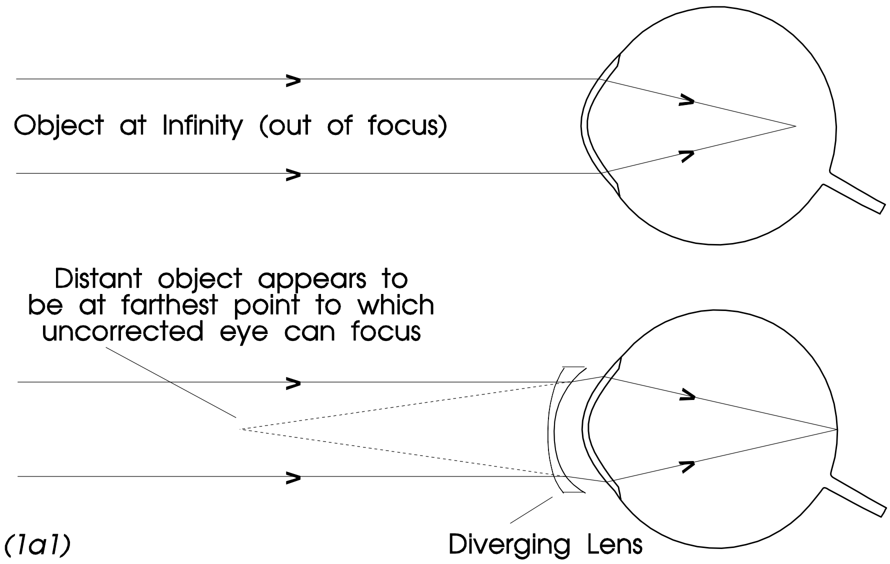
The Eye

Short Sight

The human eye is normally able to focus on objects as close as 0.25 m and as far away as infinity. If the combined power of the cornea and eye lens is too strong, the images of distant objects will be formed *short* of the retina and so will be out of focus.

Short sight is corrected by the addition of a diverging lens. The lens is chosen so that a distant object appears close enough for the short-sighted eye to focus upon.

Correction of Short Sight



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The focal length of the correcting lens may be calculated using :-

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

where u is the object distance,

v is the image distance and
f is the focal length of the lens.

Bearing in mind that the object is at infinity and that the distance to the uncorrected far point is D. Substituting :-

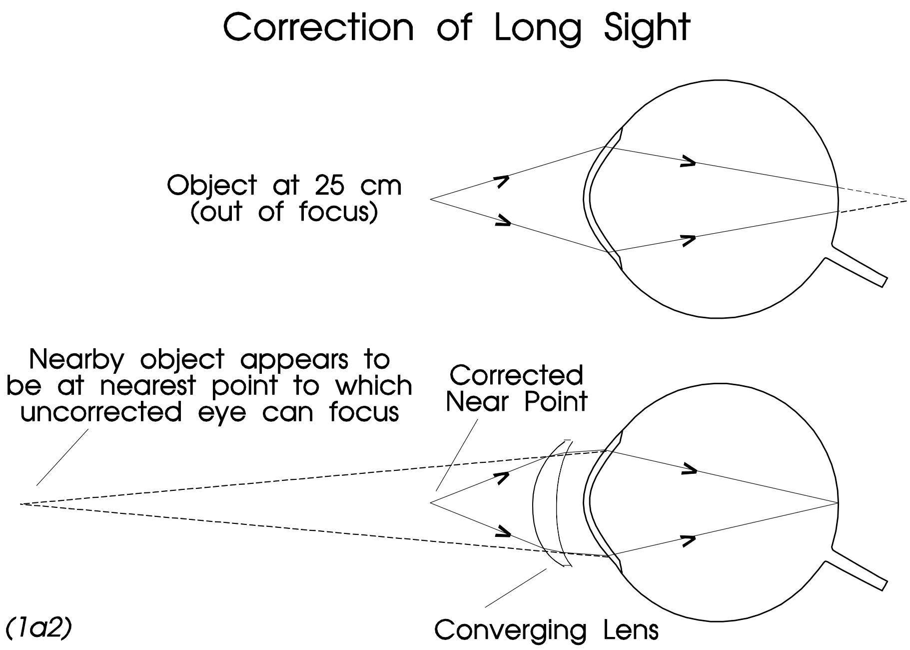
$$\frac{1}{\infty}+\frac{1}{D}=\frac{1}{f}$$

As the image created by this lens is virtual both D and f will be negative.

Long Sight

If the focusing power of the eye is too weak, the images of nearby objects will fall behind the retina (too *long*) and so will be out of focus.

Long sight is corrected by the addition of a converging lens. The lens is chosen so that nearby objects appear to be far enough away for the long-sighted eye to focus upon.



Bearing in mind that the image is made to appear at the uncorrected far point, distance D, and that the object is at the 0.25 m near point. Substituting:

$$\frac{1}{0.25}+\frac{1}{D}=\frac{1}{f}$$

As the image created by this lens is virtual, *D* will be negative.

Try holding a pair of spectacles in a beam of sunlight and look at the shadows that are cast on a wall. How can you tell if the owner is long or short sighted? You might notice that the patches of light are not symmetrical. Can you explain why?

Opticians always use a unit called the Dioptre to specify lenses. It is defined as the reciprocal of the focal length in metres of a lens. Hence a 0.25 m focal length lens has a power of 4 dioptres. This unit is practical since if a 4 dioptre lens is placed in contact with a 2 dioptre lens, the power of the combination is simply 6 dioptres. This follows from the standard u, v and f formula for lenses.

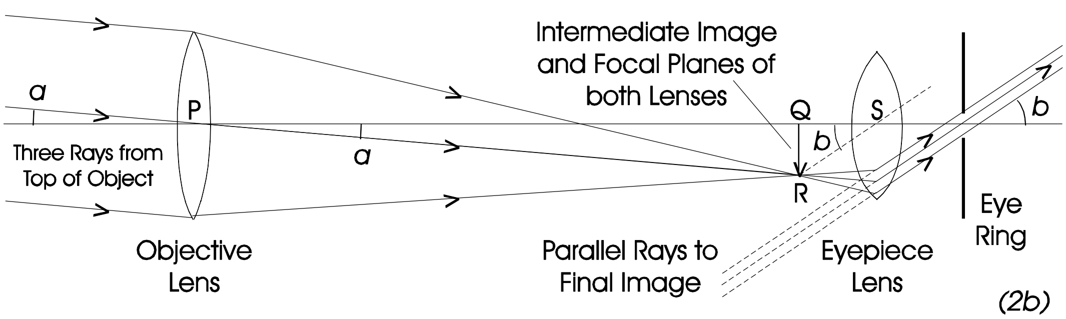
If your eye has a power of 45 dioptre and it should be 55 dioptre then your Optician prescribes a 10 dioptre spectacle lens.

Astronomical Telescope – Two Converging Lenses

Astronomical Telescope

In normal adjustment the telescope observes a distant object and forms its final image at infinity. This is achieved by allowing the objective lens to produce an intermediate image and then setting the eyepiece to be one focal length from this image. The rays enter and leave the telescope as parallel bundles. However the angle of the rays from the axis is increased, leading to angular magnification.

Astronomical Telescope



(2b)

The definition of the magnification (angular magnification) of a telescope is:

$$Magnification = \frac{Angle\ subtended\ by\ the\ image\ at\ the\ eye}{Angle\ subtended\ by\ the\ object\ at\ the\ unaided\ eye}$$

The angle subtended by the image at the eye is angle that the rays exit the telescope towards the eye – angle b.

The angle subtended by the object at the unaided eye is the same as the angle that the rays enter the telescope – angle a.

Hence the angular magnification of this telescope is just the ratio of angle b to angle a. Triangle PQR shares a side (QR) with triangle QRS. Opposite this side in each triangle are the angles a and b. In PQR the angle is a because of the opposite angle theorem. In triangle QRS the angle is b because the construction ray RS is parallel to the exit rays. Hence:

$$\tan a = \frac{QR}{PQ} \qquad \text{and} \qquad \tan b = \frac{QR}{QS}$$

but PQ and QS are the focal lengths of the objective and eyepiece lenses. Also if the angles a and b are small, the tangents may be approximated to the angles expressed in radians. Hence the angular magnification is given by:

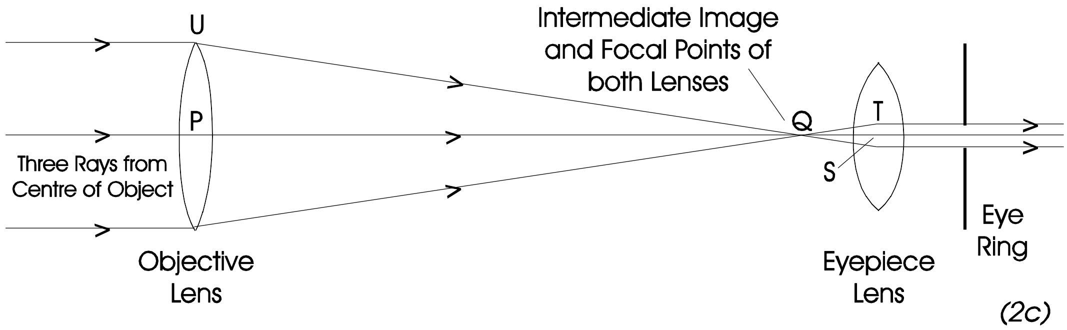
$$\frac{b}{a} = \frac{PQ}{QS} = \frac{fo}{fe}$$ the ratio of the focal lengths.

Try measuring the focal lengths of the lenses in the diagram and compare your result with the ratio of the angles measured with a protractor. Can you explain why the results don't quite agree?

Extension Material

The Eye Ring

The eye ring is a disc with a hole that is used to guide the eye to the best place to view the final image. The best position is in line with the point where the central ray crosses the axis of the telescope as it exists. The size of the hole in the plate is chosen to allow rays that have entered the telescope at the edge of the objective to just leave by the ring. This prevents the observer from seeing the inside of the telescope tube but also ensures that all the useful rays can be viewed.



(2c)

The triangles PQU and QST are geometrically similar as they have three identical angles. Hence by simple ratios, the diameter of the eye ring is given by the diameter of the objective divided by the angular magnification.

The marking 10 × 50 is typical for binoculars - which number is the magnification? What does the other represent? Try measuring the diameters of the objective and eyepiece of a pair of binoculars. Does their ratio equal the magnification as predicted? You might find the agreement rather disappointing! Try looking through the binoculars with the eyepiece about 0.25 m away from your eye. Measure the diameter of the patch of light instead of the diameter of the lens. Is this in better agreement?

Reflecting Telescopes

The Focal Point of a Concave Mirror

A curved mirror is able to bring light to a focus and so can be used to make a telescope. Rays close and parallel to the Principal Axis are reflected through the focus.

In astronomy, all mirrors are coated on their *front* surfaces, usually with aluminium but occasionally with silver.

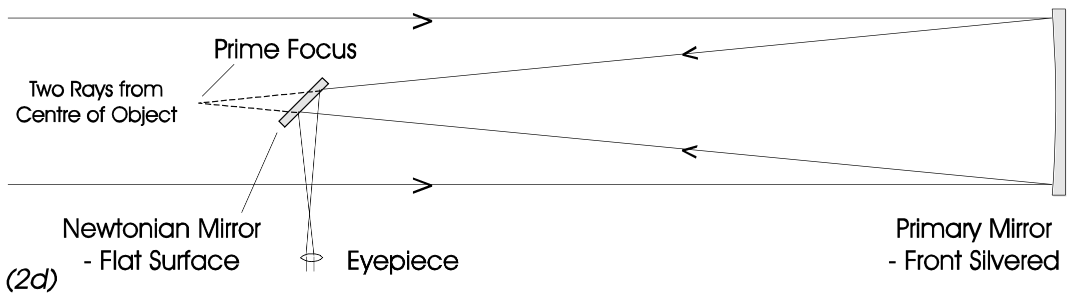
Types of Telescope

There are at least three ways in which the image can be viewed - at the prime focus in the centre of the telescope tube, at the side of the tube (Newtonian arrangement) or behind the primary mirror (Cassegrain arrangement).

The Newtonian Telescope

The prime focus arrangement is popular for direct imaging as the light only interacts with one optical component, the primary mirror. The Newtonian arrangement is popular on amateur telescopes as it is simple to construct and allows the eyepiece to be in a convenient viewing position. The Cassegrain arrangement allows complex and heavy detectors to be attached to large telescopes with comparative ease.

Newtonian Telescope

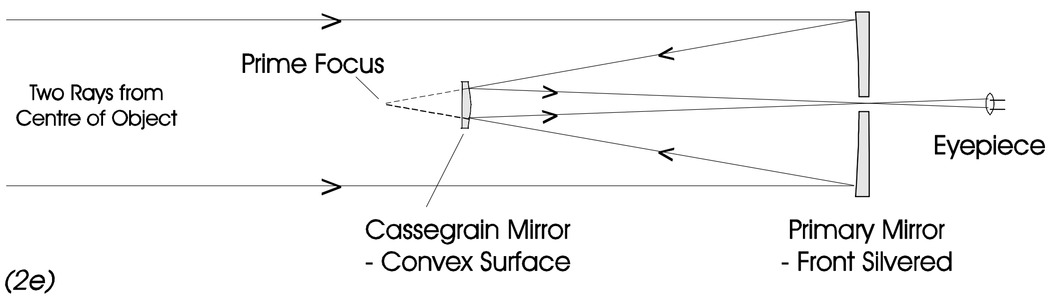


In the Newtonian telescope the rays of light are reflected by the secondary mirror so that the primary image may be observed by the short focal length eyepiece at the side of the telescope tube.

The Cassegrain Telescope

In the Cassegrain telescope the rays of light are reflected by the convex secondary mirror to form a focus behind the primary mirror. Heavy and complex equipment is often fastened there to make images and spectra of the light.

Cassegrain Telescope



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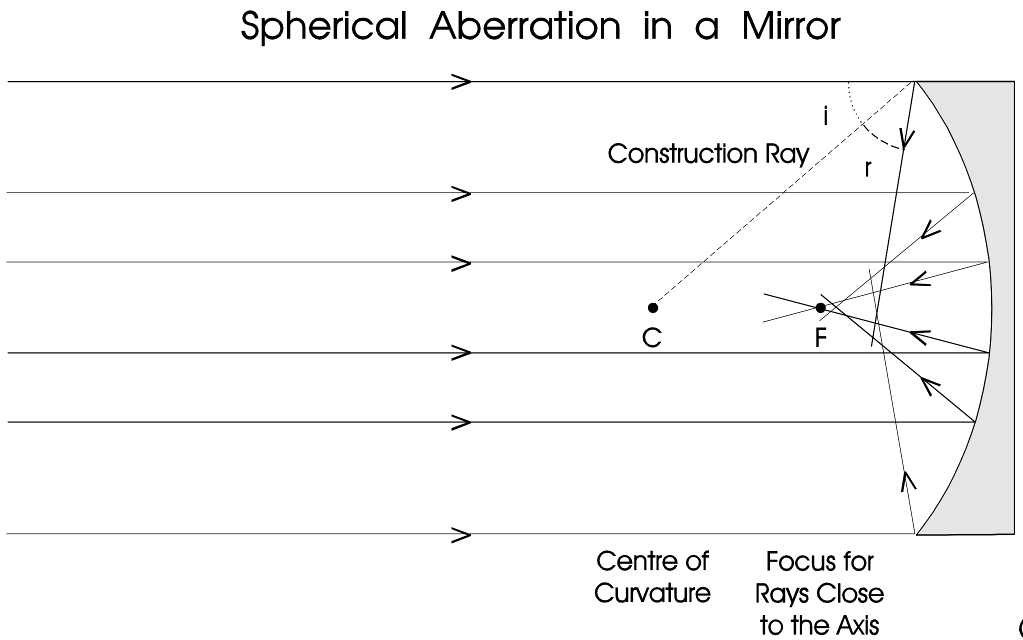
The Relative Merits of Reflecting and Refracting Telescopes

All large telescopes are reflectors. It is difficult to produce blemish-free glass plates large enough to make lenses greater than a few tens of centimetres across and such lenses would tend to sag under their own weight. Large mirrors are made possible by using the front surface for reflection and by supporting the rear of the glass at many points.

The reflectors have secondary mirrors that block some light from the source. For reflectors below 15 cm in diameter the loss of light makes the design unattractive. Small refracting telescopes have no such light loss and so dominate the small telescope market.

Spherical Aberration

Spherical aberration occurs in lenses and mirrors. Rays that strike close to the centre of the lens or mirror are accurately brought to a focus. However, rays that are far from the centre of the component come to a different focus. The diagram is built up using equal angles of incidence and reflection as shown with the construction ray.



(2g)

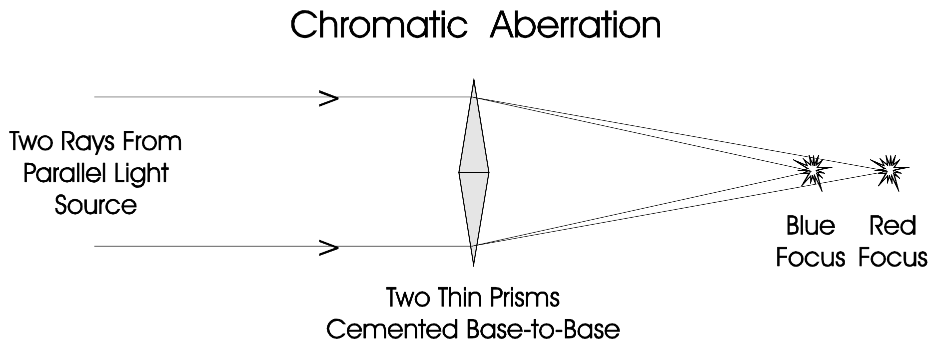
Try holding a cup of tea at the side of a strong light. Observe the Caustic Curve that represents the sum of all the various focal points.

Spherical aberration in mirrors can be eliminated for rays parallel to the optical axis if the surface is ground to a paraboloidal shape. Unfortunately objects off the optical axis produce images that suffer from coma – they look egg or comma shaped.

Lenses may be corrected for spherical aberration by having their surfaces ground to complex shapes that are often determined experimentally by ray-tracing.

Chromatic Aberration

Chromatic aberration occurs in lenses because the refractive index of glass varies slightly over the range of visible light. A simple model of a lens can be made by imagining two thin prisms cemented together base to base. Red light is refracted least and blue most. The blue image will be nearer the lens and the red image further away.



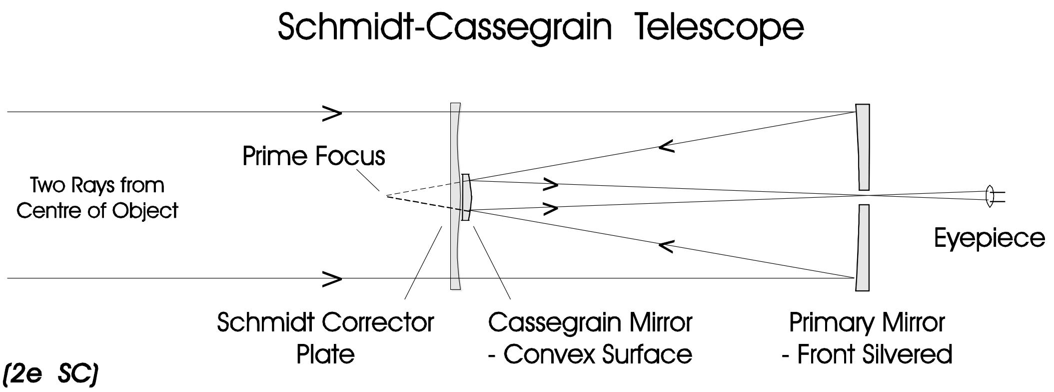
(2f)

This problem leads to falsely coloured images and a reduction of sharpness of focus in black and white photography. It may be reduced by placing two lenses of slightly different glass in contact with each other. The differences of refractive index and focal length can be chosen to cancel the aberration at two particular wavelengths and to minimise it at others. Such a lens combination is called an Achromatic Doublet. Reflecting telescopes do not suffer from Chromatic Aberration.

Extension Material

The Schmidt-Cassegrain Telescope

This design is widely used in larger amateur telescopes. It is well corrected for aberrations and has a closed tube that protects the primary mirror.



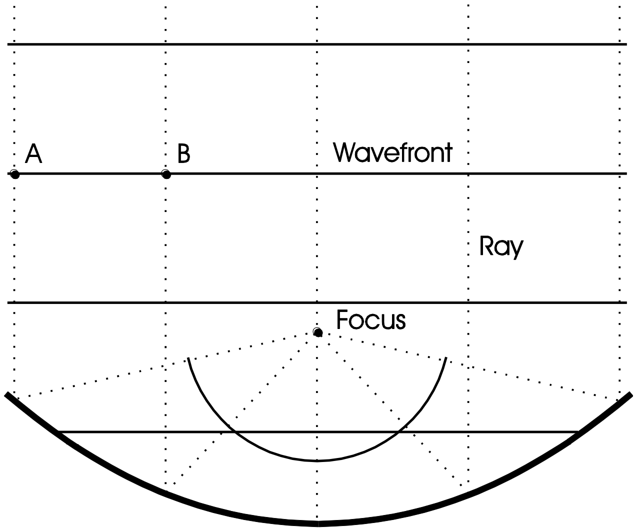
(2e SC)

Wavefronts Being Focussed by a Parabolic Mirror

When an object is very distant, the rays of light from it are effectively parallel. This corresponds to the wavefronts from it being plane (flat). The mirror must reflect each wavefront in such a way that all parts arrive at the focus at the same instant.

A parabolic surface has this property. Measure the distance from A to the surface and then to the Focus. Repeat this starting at B. Check that the distances are equal.

Radio Telescope and Satellite Dishes use this shape of reflector.



The lengths of the rays A to F and B to F are identical as all parts of each wavefront arrive at the Focus at the same instant
(PW)

Surface Accuracy of a Telescope Reflector

The reflecting surface must, as far as possible, bring all parts of a given wavefront to the focal point at the same instant. To achieve this satisfactorily the surface must be accurate in shape to better than one twentieth of the wavelength of the waves. At this level of accuracy the loss is about 16% compared to a perfect surface. At one tenth of a wavelength accuracy the loss is 50%.

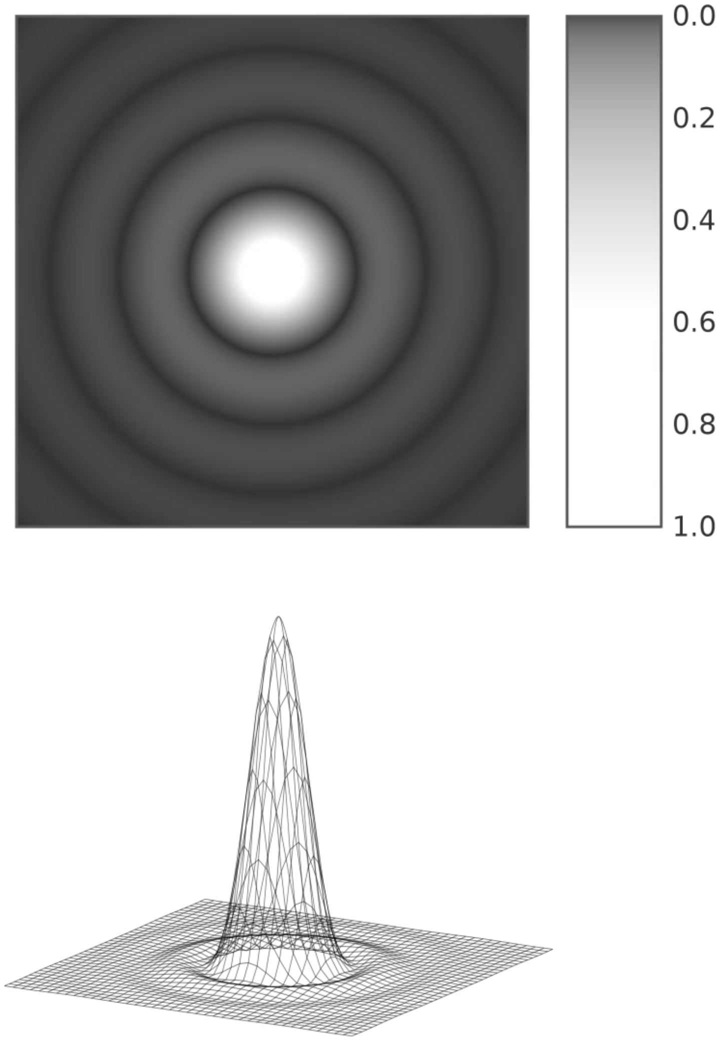
Resolving Power

Diffraction Pattern Produced by a Circular Aperture

When wavefronts encounter a barrier they are diffracted. This is often demonstrated in a ripple tank.

In the graphics below, parallel light is directed through a small circular hole (aperture). The wavefronts diffract as they pass through the aperture and then spread out to form a central maximum and rings of light that are called secondary maxima.

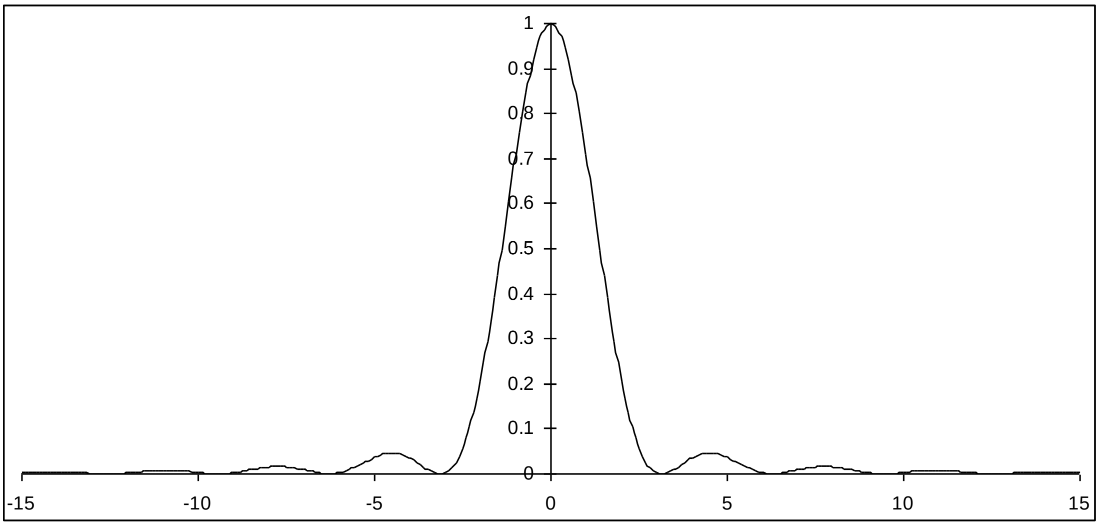
The size of the central maximum is described by the distance to the first dark ring. This dark ring is the first minimum and is clearly seen in the graphics. As well as being described as a diffraction pattern, the way the light is spread out is referred to in astronomy as the 'point spread function'.

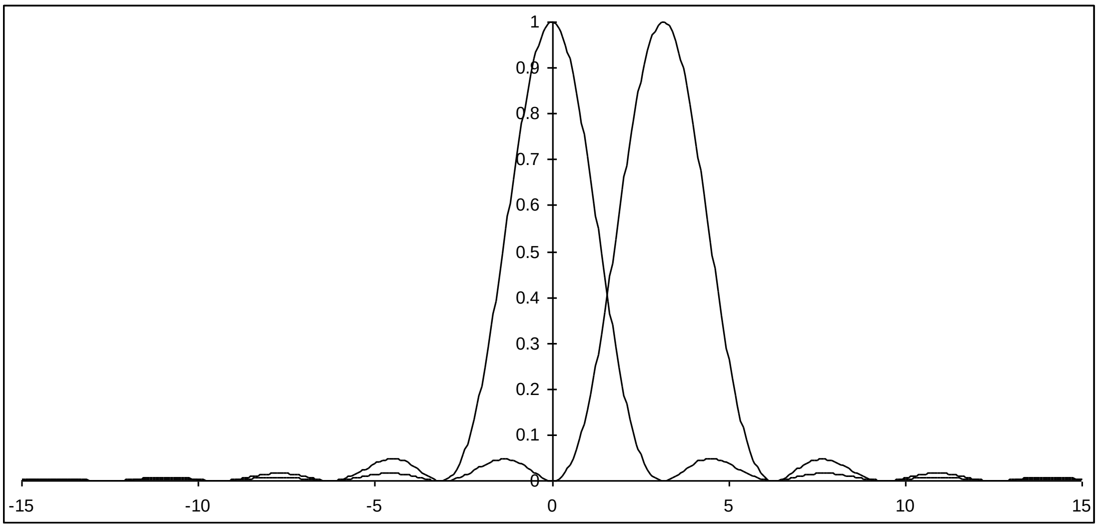


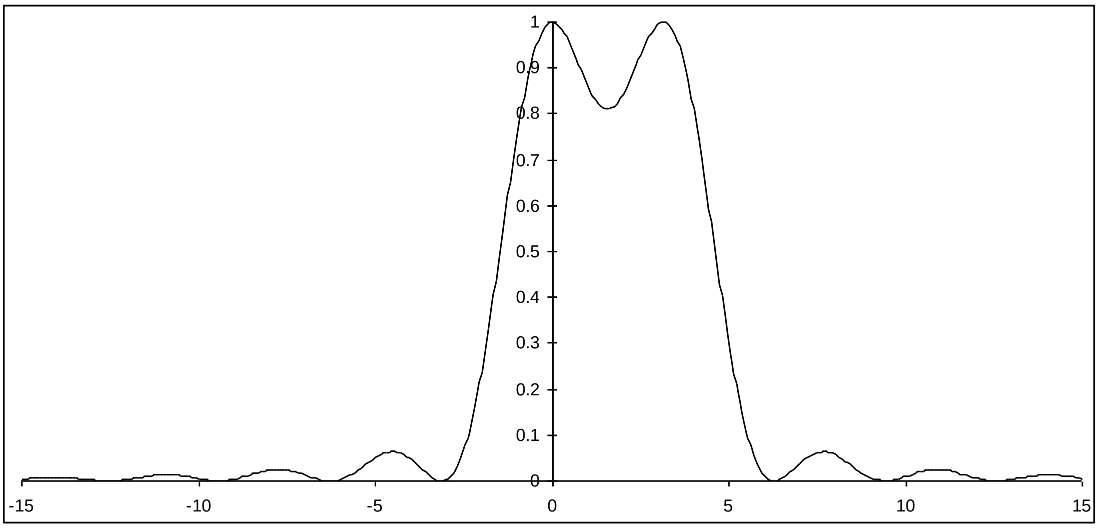
Two plots of the intensity of light that has passed through a circular aperture.
(Wikipedia – Airy disk)

The Resolving Power of a Telescope – The Rayleigh Criterion

When the light from a star enters a telescope it diffracts as shown below. If there is another star nearby, its diffraction pattern will overlap that of the first star. The sum of the intensity patterns is shown in the third graphic below. The horizontal axis is the angle from the centreline (in radians).







Lord Rayleigh (John William Strutt) devised a simple method to find if two closely-spaced images would be visible when observed through a telescope. There are three possibilities:

- The images are so far apart that they are clearly separate
- The images are quite close together but are just seen as two separate images
- The images are so close together that they appear as an oval blur.

He defined the situation where the images are just seen as separate to be when the centre of one image falls on the first minimum of the second image. This is the Rayleigh Criterion. It is the situation shown in the preceding graphic.

The angle (θ in radians) from the centre to the first minimum in a diffraction pattern is given by:

$$\theta = 1.22\frac{\lambda}{d}$$

where θ is the angle in radians,
λ is wavelength in metres and
d is the aperture diameter in metres.

For the purposes of the examination the formula is simplified to:

$$\theta \approx \frac{\lambda}{d}$$

This angle is taken as a guide to the fineness of detail it can detect in the source. The smaller the angle the finer the detail.

Detector	Theoretical Resolution in Space	Ground Based Resolution
Optical 5 m diameter	0.02 seconds of arc	1 second of arc
Eye 3 mm pupil	40 seconds of arc	≈60 seconds of arc
Lovell Telescope 76 m diameter at λ = 73 cm	40 minutes of arc	40 minutes of arc
MERLIN 134 km spacing at λ = 18 cm	0.2 seconds of arc	0.2 seconds of arc
Einstein Orbiting X-ray Telescope	5 seconds of arc	X-rays absorbed

Unfortunately, Earthbound optical telescopes can never reach their theoretical performance because of variations in the refractive index of the air caused by convection currents. This is called scintillation or twinkling and such telescopes are said to be 'seeing limited'.

Radio telescopes, particularly interferometers, can approach their theoretical performance limit and are said to be 'diffraction limited'.

As the atmospheric seeing limitation is about one second of arc, it is only worth building optical telescopes of 0.2 m diameter if resolution is the only consideration. With greater diameters no finer detail is observed, however the telescopes act as a better light-buckets! This allows more observations to be done per night as the more light, the shorter the exposure time required. Looked at another way, a large telescope can see fainter, and by implication, more distant objects if it follows them round the sky during the night!

With longer wavelengths of perhaps 10 μm, a 2 m infra-red telescope is matched to the limit above and so is 'diffraction limited'. Also the longer the wavelength, the less scintillation is a problem.

Much of the problem of scintillation can be removed by using Adaptive Optics. A small mirror is placed in the path of the light leaving the telescope. The mirror is flexible and behind it are hundreds of tiny electromagnets. A sophisticated computer system senses the scintillations and continuously changes the shape of the mirror to cancel them out.

Extension Material

Resolution of the Human Eye

The resolution of the eye depends on the size of the diffraction pattern on the retina and on the spacing of the rods and cones. Two points of light are seen distinctly if there is at least one dark cell between the two illuminated ones. The spacing of the cells in the retina just about matches the size of the diffraction pattern - a wonder of evolution!

Try placing two small clear lamps 10 centimetres apart in the window of your laboratory and then walk away from them until the two points of light merge into one image (several hundred metres). Measure this distance and calculate the angle subtended by the lamps. Now measure the diameter of the pupil of your eye and calculate the angle to the first minimum of its diffraction pattern using the formula at the start of this section. You will be amazed at the agreement. You could also look up the spacing of the cones in the retina and check that value for agreement - an accident of evolution? (The retina contains about 3×10^6 *cones and about* 1×10^8 *rods. The spacing of the cells on the retina is about* 2.5×10^{-6} *m.)*

Surface Accuracy of a Telescope Reflector

Energy from a distant object arrives at the telescope as plane wavefronts. The curved reflector focuses these waves onto a detector. To do this well the surface of the telescope must reflect the wavefront so that all parts of it arrive at the detector in phase. To achieve this satisfactorily the surface must be accurate in shape to better than one twentieth of the wavelength at which it is intended to observe. At this level of accuracy the loss is about 16% compared to a perfect surface. At one tenth of a wavelength accuracy the loss is 50%.

Charge-Coupled Devices (CCDs)

The First Detectors

Humans first saw the planets and stars with their naked eyes. Then in 1609 Galileo Galilei looked at the sky through a simple two-lens telescope. Within a year he had seen and sketched the craters and plains on the Moon, that Venus showed phases like the Moon, that Jupiter had four moons of its own and that the Milky Way was a mass of stars.

It is interesting to note that in 1608, Thomas Harriot acquired a telescope from Holland. In early 1609, before Galileo, he became the first astronomer to use a telescope. He observed and sketched the Moon. However Harriot did not publish his work so he is forgotten and Galileo is remembered!

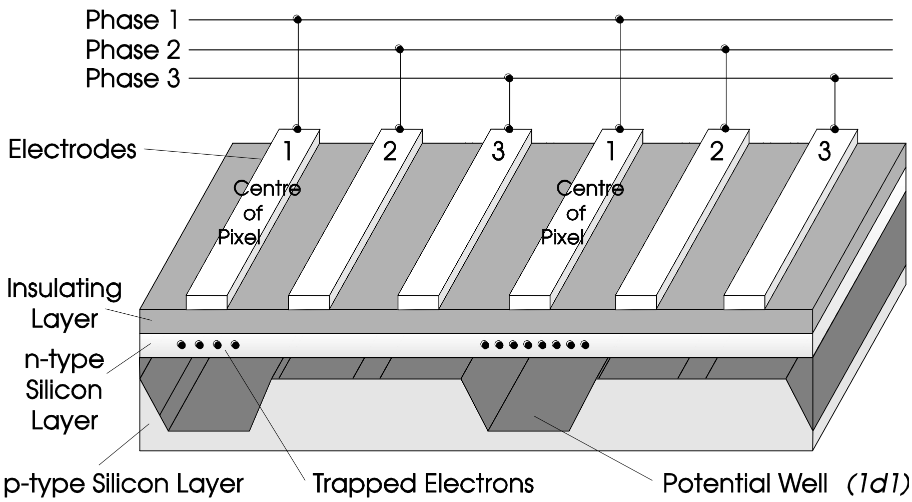
The first astronomical photographs were taken by Louis Daguerre. In 1839 he took images of the Moon but none survive as his laboratory burned to the ground soon after, destroying all his written records and the bulk of his experiments!

The CCD

The CCD has superseded the eye and photographic plate in astronomy. It has revolutionised optical observations. The light collected by a professional telescope can be used to create an image on a CCD or it can be directed onto a diffraction grating to produce a spectrum that is then recorded on a CCD.

With the eye there is no possibility of storing the incoming photons to improve a faint image. However a CCD will collect photons for as long as required. The longer the exposure the denser the image (until saturation). This process is called 'integration' (adding up).

Charge-Coupled Device



(1d1)

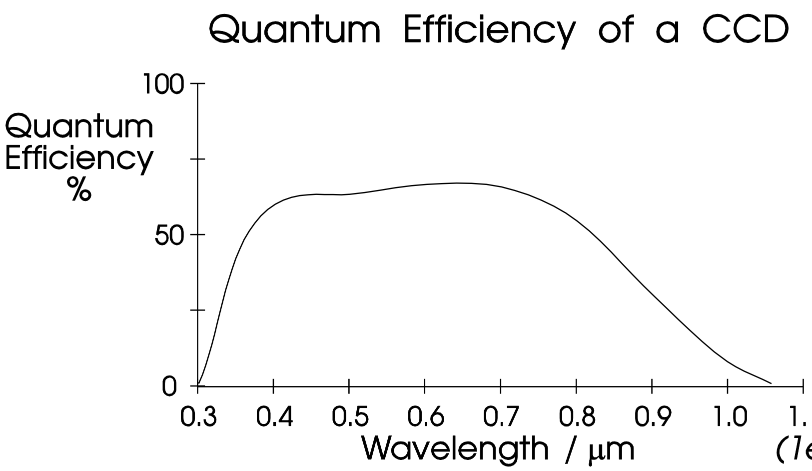
As light falls on the pixels, individual photons liberate electrons from the atoms in the lattice. These electrons are trapped in 'potential wells', one for each pixel. After a few minutes of exposure on a large telescope sufficient charge will have collected in each pixel for it to be successfully 'read out'. The charges are moved along to the ends of their respective rows in a systematic manner, in a way similar to that used to control the running lights at Blackpool Illuminations. In a repeated sequence from one to three, each control wire is made more positive than the other two wires. This draws the electrons to the more positive electrode and so moves the charges from left to right. At the ends of the rows the electrons are sensed using a specially designed amplifier.

The quantum efficiency is perhaps greater than 70% (c.f. film at between 1% and 4%) This allows images to be obtained more rapidly, thereby increasing the number of images collected per observing session. Alternatively the greater sensitivity can be used to see fainter objects by tracking for long periods.

A further advantage of the CCD is its Dynamic Range. It is much better than film in this respect; on the same exposure, detail can be seen in the faint spiral arms of a galaxy as well as in its bright nucleus. This advantage is accompanied by an excellent linearity of response. From the noise floor of a few electrons all the way up to the maximum charge per pixel of typically 100 000 electrons the chip is linear in response to the incoming light.

CCDs are often cooled to –100 °C to reduce random emission of electrons. A dark current (random effects) as low as 5 electrons per hour in each pixel can be expected!

A typical response curve for a thinned and coated CCD is shown below.

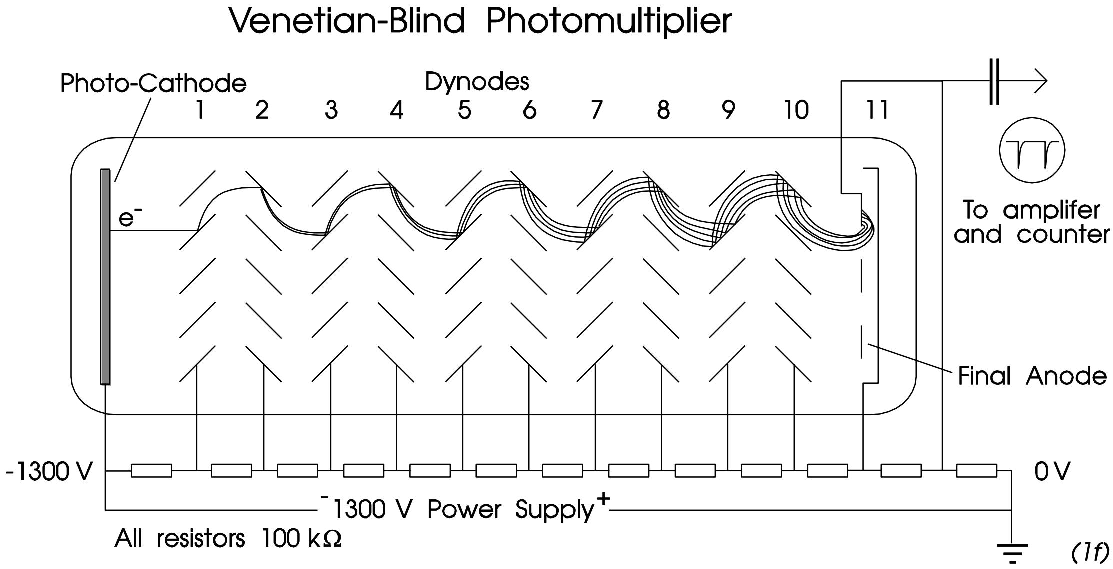


(1e)

Modern CCDs are large silicon wafers (40 mm x 40 mm) divided into an array of pixels, typically 4 096 by 4 096, giving a pixel width of about 10 µm. Each CCD might cost £1000!

Extension Material

Photomultipliers – Counting Single Photons



(1f)

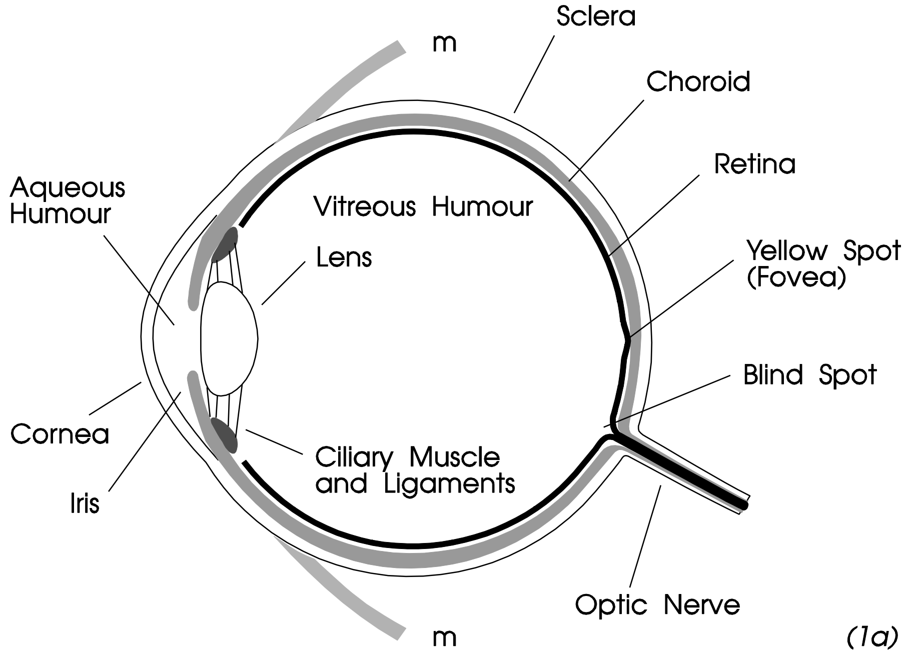
An incoming photon strikes the photo-cathode knocking out an electron. The electron is then accelerated towards another surface from which it displaces several more electrons. This process is repeated up to ten times so that the original photon gives rise to an easily measurable pulse of electrons. These devices may be used at very low light levels as photon counters and are *much* more sensitive than photographic films.

In the vacuum tube above there are 10 stages of multiplication, each dynode more positive than the one before. If, say, the number of electrons emitted for each electron striking a dynode is 5 then the output pulse would contain $5^{10} = 9.8 \times 10^6$ electrons. If the final anode was of capacitance 10 pF then the pulse would be about 0.2 volt. *Can you show this using $e = 1.6 \times 10^{-19}$ C and $Q = CV$?*

The photocathode efficiency is between 10% and 20% and its response as a function of wavelength is similar to that of the photocell. Note that the device does not produce an image, only a point reading. They can be accurately calibrated and so can be used to compare the intensities of stars.

When no light falls on the device some pulses will still be observed. They are probably caused by radioactive decays in the tube or by β-particles or γ-rays entering the tube causing ionisation. Also thermal electrons can be ejected from the cathode (cooling to –25 °C or below can help) and stray positive ions may also cause cascade events. With cooling the stray pulse count rate can be reduced from about 20 to as little as 1 per second.

The Human Eye - + (Blind Spot Test)



(1a)

The main focusing component is the curved cornea. The lens is submerged in the humours that are of similar refractive index to the material of the lens. The lens 'fine tunes' the formation of the image on the retina as the ciliary muscles change its shape.

The eye is rotated by muscles attached at the points 'm' until the central part of the image falls on the yellow spot where the majority of colour sensors are located.

The iris adjusts the aperture (pupil) of the eye for variations in brightness. A variation from about 2 mm to 6 mm diameter gives a 1 to 9 control over brightness of the image (ratio of areas).

Look at your eyes in a mirror. Shade them from the light and watch the pupils dilate. Notice the time delay.

The eye is able to function over a much wider range of brightness than 1:9. The rest of the sensitivity variation is due to chemical concentration changes in the rods and cones of the retina. The range can be as large as 10^9 or 10^{10} times with the response being proportional to the logarithm of the intensity.

The logarithmic response is simple to understand. Your eye compares the power falling on the retina from various objects as follows. Imagine three lamps of equal size - A and C - have powers of 5 and 45 units. Lamp B will seem to be half way in brightness between A and C if it has a power of 15 units. Put another way, the ratio of power A to B is 1:3 and so is the ratio of B to C. (There is nothing special about the ratio of 3 – any number will do.) The logarithmic name comes from the fact that the logarithm of the ratio is the same in both cases (obviously!!). You can try this with real lamps but it is not very easy or very convincing!!

The sclera provides the physical support for the eye. The choroid, which is dark brown because it is rich in blood vessels, provides nutrition and absorbs scattered light. It is seen as red-eye in flash photographs.

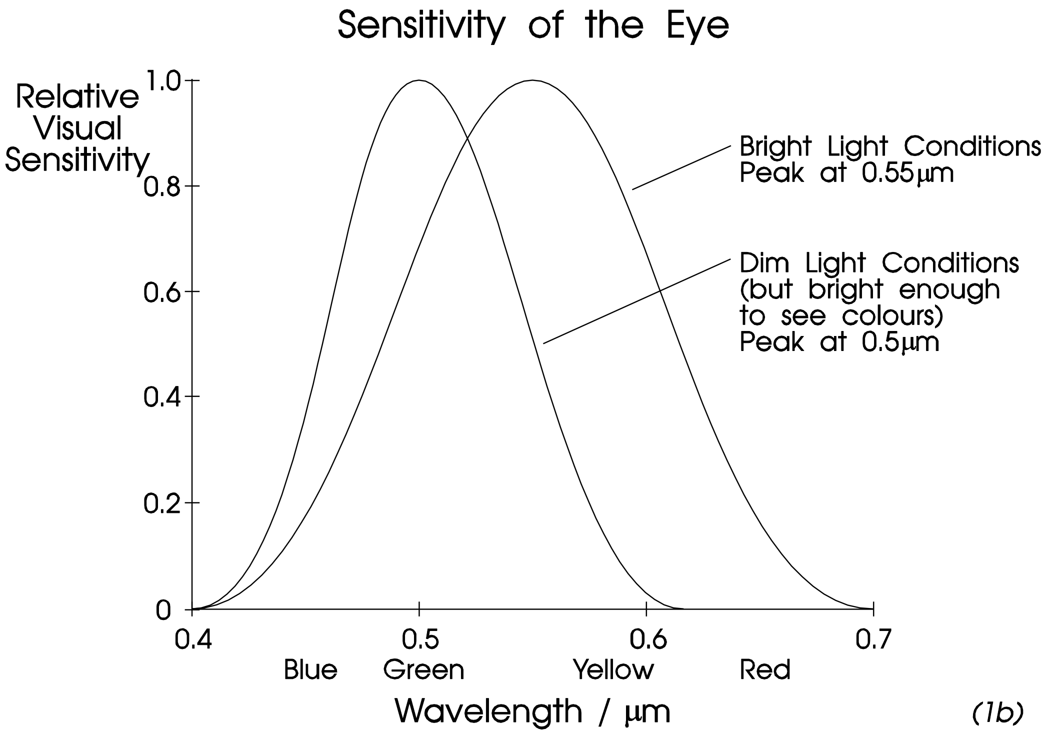
The whole of the retina is populated by rods that can sense only light and dark but are able to do so down to very low light levels. The fovea is additionally populated by cones that are sensitive to the colour of the incident light but which are not very sensitive to low light levels. This is why you first lose the colours in your surroundings as it goes dark. It also explains why most stars look white, as they are not bright enough to trigger the less sensitive cones.

The trick of 'averted vision' may be used to view faint stars. Look at the faint star of interest and then deliberately switch your view to a blank part of the sky a little way away (a few degrees) from the target. The faint star suddenly seems brighter! This is because its light is now falling on the rods at the side of your yellow spot and so causes a greater reaction than it did on the less sensitive cones. The Seven Sisters of the Pleiades make a good target for this experiment.

Check out the distribution of cones in your eye by holding a brightly coloured pen at arm's length ahead of you. Stare ahead whilst slowly moving your arm to the side, keeping it outstretched. At about 45° the pen will seem to turn black. If you move it round to about 80° and hold it still, it will disappear. If you wave it slightly it will reappear but will still be without colour. This is because the edges of the retina are wired-up to respond only to movement - possible danger.

The quantum efficiency of the eye is such that only about 1% of the incoming photons produce a response. The threshold of detection for the eye occurs when it receives as few as 1000 photons per second from a point source.

It is interesting to calculate the number of photons per second entering the eye from a distant street lamp. Consider a 100 W sodium lamp, working at 50% efficiency, emitting photons of wavelength 600 nm, at a distance of 40 km. Assume the pupil of the eye is 2.5 mm radius. Would you expect to see the lamp at this distance?



The graph shows the response of the eye. In bright light the cones are mainly responsible for detection. Their colour response is centred in the middle of the visible range although the eye is clearly more sensitive to yellow/green than to red or blue. As the intensity falls, so the red response falls away, giving rise to red objects that look almost black in poor light - the Purkinje Effect.

It would be nice to say that one curve represents the response of the cones and one that of the rods but the processing in the brain has a major effect on our perception of colour.

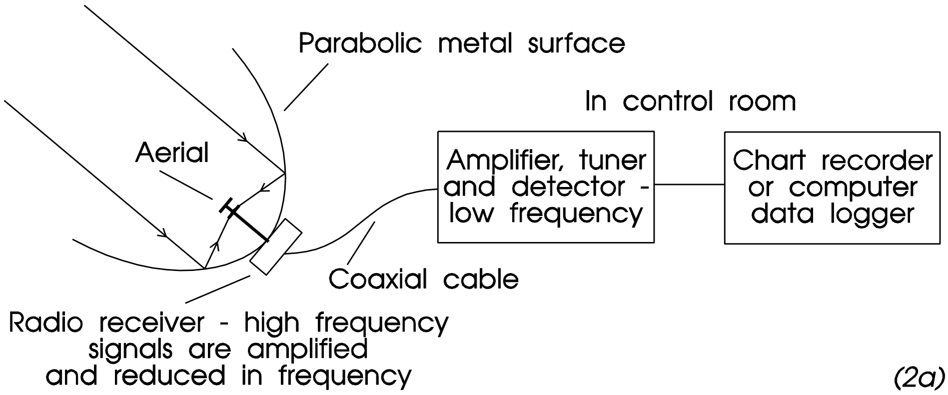
The eye adapts to low light levels given sufficient time. The cones become about 40 times more sensitive taking at least 7 minutes whereas the rods become several hundred times more sensitive but take at least 20 to 30 minutes. *Can you say why is red light used in operational submarines and to read star charts? The sensitivity graph helps with the explanation!*

You can get some idea of the time required for the chemical concentrations in your retina to recover by looking briefly at a bright light. Look away and close your eyes. Admire the various colours that the after-image passes through over at least a minute!

Non-optical Telescopes

Energy from a distant object arrives at a telescope as plane wavefronts. The curved reflector focuses these waves onto a detector. To do this well the surface of the telescope must reflect the wavefront so that all parts of it arrive at the detector in phase. To achieve this satisfactorily the surface must be accurate in shape to better than one twentieth of the wavelength at which it is intended to observe. At this level of accuracy the loss is about 16% compared to a perfect surface. At one tenth of a wavelength accuracy the loss is 50%. The telescope will usually be driven by motors so that it tracks the source as the Earth turns.

Radio Telescopes



(2a)

The parabolic metal surface is used to focus the plane waves onto the aerial. Immediately behind the aerial is a preamplifier that boosts the very faint signal before it is sent down a cable to the control room. There the signal is further amplified and the required band of frequencies selected. The signal is then detected and fed to a recording device. In the early days of radio astronomy this would have been a chart recorder. Nowadays a computer is used and detailed information about the spectrum of frequencies being received is be recorded. The spectrum can then be used to identify the chemical elements in the source and also its velocity.

The power of the signal focused onto the aerial is proportional to the collecting area of the telescope. The 76 metre diameter Lovell Telescope would produce a signal over 6 000 times more powerful than a one metre diameter domestic satellite dish when directed at the same source.

Image Formation

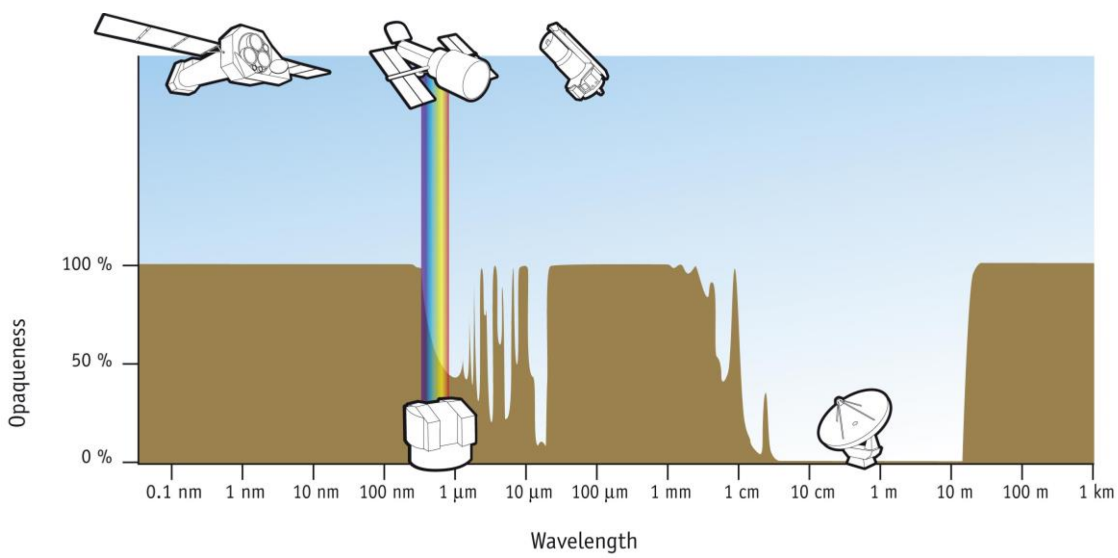
An optical telescope can produce an image from its mirror system whereas a radio telescope focuses all its incoming energy onto the aerial and no image is formed. If an image is required from a simple radio telescope, the dish must be scanned over the source many times, to build up the picture as a series of strips. In modern systems, such as MERLIN at Jodrell Bank, several radio telescopes are linked to form an interferometer that can, after much computer processing of their signals, produce images of great detail.

This trend is seen in the new ALMA (Atacama Large Millimetre Array) which will have 64 radio dishes linked to a very powerful computer system. The array is likely to be able to discover the types of molecules in the atmospheres of newly-discovered exo-planets. This will indicate the likelihood of life on those planets.

Telescopes Above the Atmosphere

Our view of the Universe is severely limited by the Earth's atmosphere. On the surface of the Earth, UV is filtered out by the ozone layer and IR is filtered out by water vapour and carbon dioxide. Gamma and X-rays ionize atoms in the top of the atmosphere and never reach the surface.

The illustration below shows that visible and radio observations can be made from the surface of the Earth but gamma ray, X-ray, ultraviolet and infrared observations have to made from space.



Absorption of electromagnetic waves by the Earth's atmosphere. F. Granato (ESA/Hubble)

Gamma Ray Telescopes

The Swift satellite is dedicated to the study of gamma-ray bursts (GRBs). It continuously monitors space for GRBs and as soon as one is detected it “swiftly” swings round to observe in gamma-ray, X-ray, ultraviolet, and optical wavebands.

Gamma-ray bursts (GRBs) are flashes of gamma rays associated with extremely energetic explosions in distant galaxies. They are the most luminous electromagnetic events in the universe. Bursts can last from ten milliseconds to several minutes, although a typical burst lasts 20-40 seconds. The initial burst is usually followed by a longer-lived "afterglow" emitted at longer wavelengths. The burst is thought to be a narrow beam of intense radiation released during a supernova event, as a rapidly rotating, high-mass star collapses to form a neutron star, quark star or black hole.

X-ray Telescopes

The European Space Agency (ESA) launched and operates the satellite XMM Newton. The satellite is able to detect of X-rays from Solar System objects, star-forming regions in our galaxy, supernovae, distant clusters of galaxies and supermassive black holes.

X-ray sources are often associated with binary stars, one component of which is a neutron star or black hole. The X-rays are generated when the neutron star pulls material from its partner and this material is accelerated to very high velocities before collisions occur.

Ultraviolet Light Telescopes

India intends to launch Astrosat in 2012. It will have visible, UV and X-ray telescopes.

Visible Light Telescopes

The Hubble Space Telescope is able to work without the limitations of the atmosphere. Its mirror functions up to its diffraction/surface accuracy limits and is not distorted by gravity. It is due to be replaced by the James Webb Space Telescope in the next few years.

The Kepler spacecraft is designed to discover Earth-like planets orbiting other stars in our galaxy. It is concentrating on a small region of the sky between Vega and Deneb were it is observing 140 000 stars in its 4-year mission. It is searching for slight dips in the brightnesses of the stars. The dips could be caused by a planet passing in front of its star – a transit. However, this will be a very small effect. When Jupiter transits the Sun it produces a 1% dip in brightness. For an Earth-sized planet in the “Goldilocks Zone” there will be a 0.01% dip in brightness of its sun for a few hours, once every year – assuming we are in the orbital plane of the planet.

This is why the mission needs to look at so many stars for so long. At the end of 2011, Kepler has found 2 Earth-like planets. However, as the data accumulates, many more are expected.

Infrared Light Telescopes

ISO, the Infrared Space Observatory, was launched by ESA. Its telescope was cooled to 1.7 Kelvin using superfluid helium. It observed between wavelengths of 2.5 to 240 micrometres and observed 30 000 sources in great detail. Many were star-forming regions in our galaxy, such as the Orion Nebula. The dense dust clouds prevent us seeing into the nebula whereas the IR, with its longer wavelength, can pass through them easily.

Space Probes

Space probes have visited the Moon, the Planets and Comets (Voyager, Giotto, Magellan, Mariner and Pioneer). Search the web for "Solar System Probes".

Extension Material

Mesh Dishes

Some dishes are made of wire mesh to reduce their weight, in which case the holes should be smaller than $\lambda/20$ if the surface is still to be an efficient reflector.

Try using a 3 cm wave apparatus to test the reflection efficiencies of wire netting of various degrees of fineness. If wire netting is hard to find, make a reflector by laying metal rods or wires at right-angles to each other on the bench. Try seeing how much energy is reflected and how much passes straight through. Also try reflecting the microwaves from slightly crumpled aluminium foil to check the $\lambda/20$ *criterion for the flatness of mirrors.*

It has been stated that the holes in a dish reduce the wind resistance but this is not the case, particularly not at high wind speeds! *Try suspending a fine mesh on two strings about one metre long in front of a fan and then repeat the experiment with the mesh covered with cling film. Are the wind resistance forces similar?*

Radar Astronomy

In the years immediately following the Second World War, Bernard Lovell set up surplus radar equipment at Manchester University in Oxford Road near the city centre. He was seeking echoes from cosmic ray showers but instead detected the ionised trails left by meteors burning up in the top of the atmosphere. Unfortunately the interference from the overhead electrical equipment of the passing trams was so great that he was forced to look for a radio-quiet site. The University offered a field in Cheshire at their Botanical Research Station – Jodrell Bank!

Ground-based studies of near-by planets were very popular in the 1950s and 60s but have been somewhat overtaken by space probe investigations. Good quality maps of the cloud covered surface of Venus were made and its period of rotation measured. Using long wavelength radar, sub-surface maps of the Moon proved of interest before space probes were able to soft land.

Distance and rotation have been measured for all the inner planets and these results have improved the estimate of the Astronomical Unit (AU).

More recently nearly 70 asteroids have been probed by radar and some interesting discoveries made. In particular an asteroid has been found which is such a good reflector that its surface must have a very high proportion of metals. This asteroid passed within 20 Gm of the Earth in 1986! Using a large radio-telescope in California, signals were directed at the object at 3.5 cm wavelength. The echoes were received by the 27-dish Very Large Array in New Mexico where detailed maps of the asteroid were produced as it tumbled through space.

In 1991 workers at Caltech directed a 500 kW, 3.5 cm wavelength pulsed radar at Mercury for 8 hours. The faint echoes were picked up by the VLA and seemed to show that the poles of the planet were made of highly fractured ground containing water ice. An unexpected result considering the 430 °C temperature of its daylight side!

The orbiting radar probe Magellan has mapped the surface of Venus in great detail, despite the planet's complete cloud cover.

Classification of Stars

Classification by Luminosity

The total power, the Luminosity (L), emitted by a spherical star, assumed to be a perfect black body, is given by multiplying the surface area by the brightness:

$$L = A \times \sigma T^4 \quad \text{where A is the surface area of the star.}$$
$$= 4\pi r^2 \times \sigma T^4 \quad \text{where r is the radius of the star in metres.}$$

Can you use this formula to show that if about 1.5 kW of power arrives from the Sun on each square metre of the Earth's disc, then the Earth's temperature has to be about the freezing point of water?

The inverse square law for radiation stems from the fact that if a point source is surrounded by an imaginary sphere then all the radiation will have to pass through it. If the sphere's radius is doubled then its area is quadrupled and so the power received per square metre by the observer of the source will be quartered as viewed from the surface of the sphere:

Area of sphere of radius $r = 4\pi r^2$
Area of sphere of radius $2r = 4\pi \times (2r)^2 = 4 \times 4\pi r^2$

The intensity (I) is defined as the power per square metre (W m^{-2}) arriving at the observer. Using the imaginary sphere argument, the intensity of a star of luminosity (L) is given by :-

$$I = \frac{L}{4\pi D^2}$$

where I is the intensity in W m^{-2} at the observer,

L is the luminosity in W and
D is the distance of the source in m.

If the intensity of the source can be measured and its luminosity estimated then its distance may be calculated. *Look in the chapter of Useful Data for the values of the Solar Constant and Luminosity of the Sun. Use them to estimate the distance of the Sun.*

Classification by Apparent Magnitude

Ever since the time of the ancient Greek astronomers, Hipparchus and Ptolemy, the visible stars have been divided into six magnitude groups as observed from Earth. The brightest are called first magnitude and the ones that are just visible to the unaided eye are called sixth magnitude. The apparent magnitude depends on how brightly a star is shining and on how distant it is from the Earth; in other words, how bright it appears to the observer. In 1827 John Herschel deduced from his measurements that a five magnitude difference corresponded to a factor of 100 in the Intensity (power per unit area) detected by the observer.

In 1856 Norman Pogson proposed that all the magnitude steps should be the same ratio, one to another. This leads to the ratio being:

$$(100)^{1/5} : 1 = 2.512 : 1$$

Confirm this result by multiplying 2.512 by itself five times!

Classification by Absolute Magnitude

To allow stars at different distances to be compared, it is convenient to adjust their Apparent Magnitudes to what they would have been if the stars were placed at the Standard Distance of 10 parsecs (The parsec is defined elsewhere).

In the following, *upper case* letters refer to the star at 10 parsecs and *lower case* letters to the star at its true distance.

Now $m - M$ = difference in magnitudes
and is called the 'Distance Modulus',
m is the Apparent Magnitude and
M is the Absolute Magnitude.

Then $$2.512^{m-M} = \frac{I_{10}}{I}$$

where I is the intensity at distance d and
I_{10} is the intensity at 10 parsecs.

N.B. The index is $m - M$ and not $M - m$, because fainter stars have a larger magnitude. For bright stars $m = 1$ or 2, for faint visible stars $m = 5$ or 6.

As mentioned earlier, the luminosity (L) is defined as the power (in watts) radiated by the star. Hence the intensity (I), being the power per unit area at the observer, is measured in watts per square metre (Wm^{-2}).

Now the intensity (I) will be determined by the inverse square law.

Hence $I \propto \frac{1}{d^2}$ where d is the distance to the star in parsecs.

$\therefore \frac{I_{10}}{I} = \left[\frac{d}{10}\right]^2$ and $\frac{I_{10}}{I} = 2.512^{m-M}$

where I is the intensity at distance d and
I_{10} is the intensity at 10 parsecs.

Equating the two expressions for intensity:

$\therefore \left[\frac{d}{10}\right]^2 = 2.512^{m-M}$ then taking logs base 10

$$\therefore 2\log d - 2\log 10 = (m-M)\times\log 2.512$$

$$\therefore 2\log d - 2\times 1 = (m-M)\times 0.400$$

$$\therefore m - M = (5\log d) - 5$$

$$= 5\log\frac{d}{10}$$

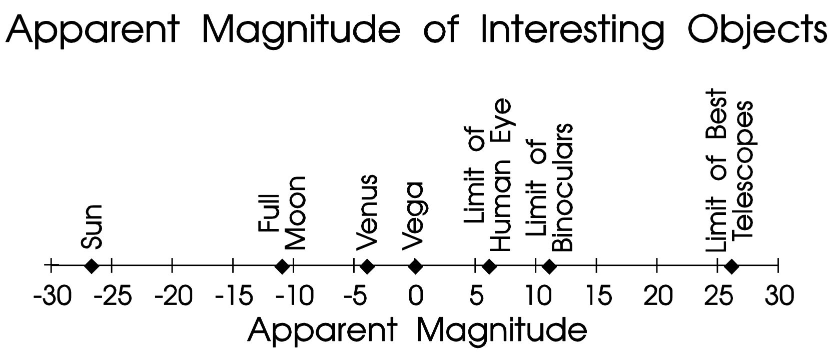
An alternative form, often found in Astronomy books is:

$$\therefore m - M = (5\log_{10} r) - 5$$

The proof of the formula is *not* required, only the result and its applications.

The formula is used as follows. If a star is of a known type then its absolute magnitude can be determined (i.e. Cepheid Variables etc. or using the HR diagram). Its apparent magnitude can be measured by observation and so its distance from the Earth in parsecs can be calculated using the formula. However, great care must be taken when applying this formula to stars that are observed through interstellar dust clouds as much light can be absorbed so making the distance estimate very inaccurate.

N.B. Magnitudes may be negative. The apparent magnitude of the Sun, which is very bright, is –26.7!! For very distant and/or faint objects the magnitudes may be large. Faintest observable object with large telescope is about +26.


Apparent Magnitude of Interesting Objects
Sun
Full Moon
Venus
Vega
Limit of Human Eye
Limit of Binoculars
Limit of Best Telescopes
-30 -25 -20 -15 -10 -5 0 5 10 15 20 25 30
Apparent Magnitude
(6a)

Classification of Stars

Classification by Temperature

Stars can be classified by their colour.

The names Red Giant and White Dwarf are in common use. In astronomy the names Blue Giant and Brown Dwarf occur frequently.

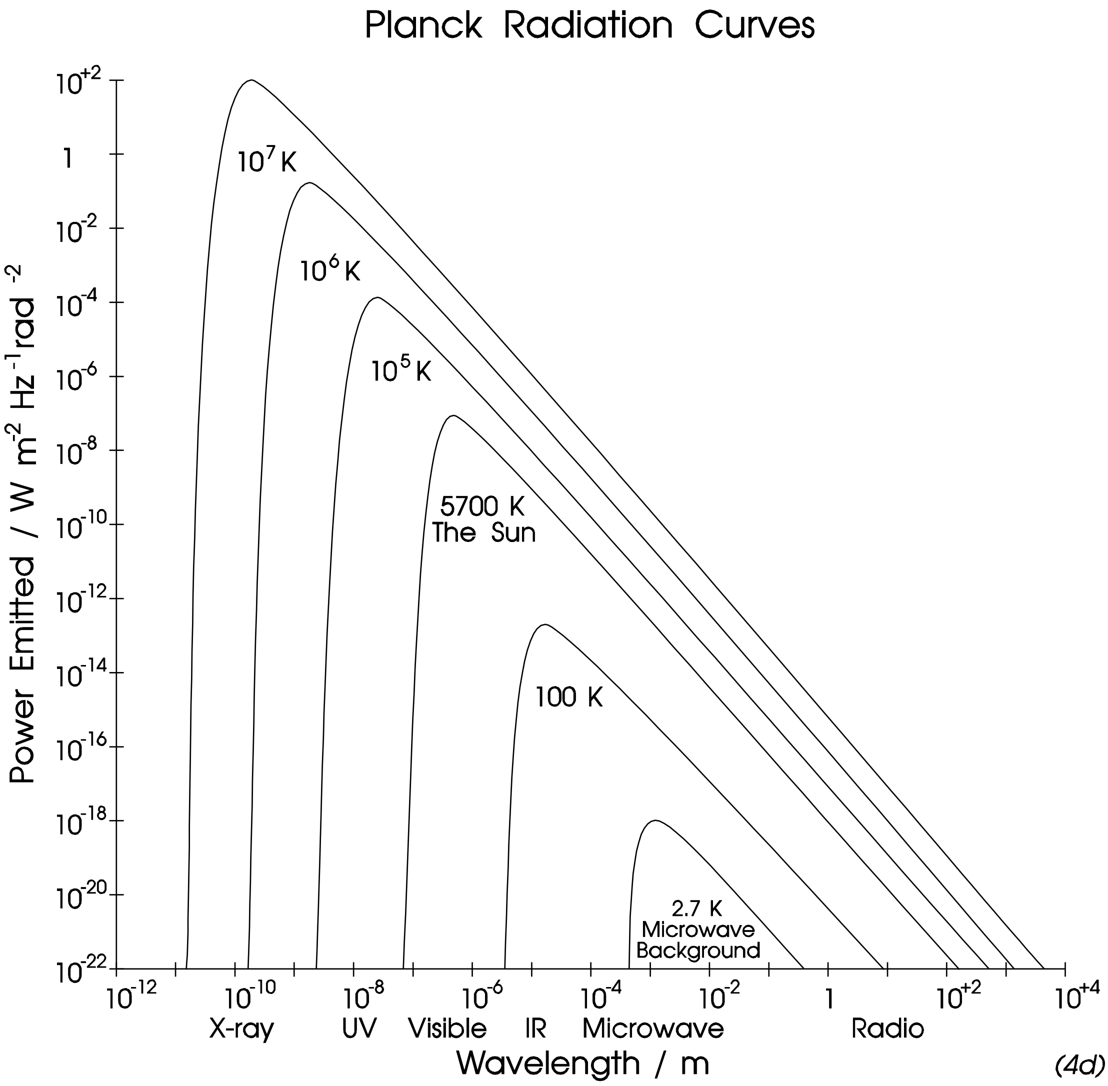
These terms are based on the temperature of the surface of the star.

As the temperature of an object increases so the radiation it emits changes.

- A cool star will only emit infrared radiation.
- At about 3 000 K it starts to emit red light.
- At 6 000 K it emits equally across the visible part of the spectrum and so looks white.
- At 12 000 K it emits more blue light than other colours and so looks blue (it will be emitting a considerable amount of ultra-violet light as well).

Blackbody Radiation

A blackbody is defined as an object that absorbs all radiation falling onto it – a perfect absorber. It can be shown that such a body must also be a perfect radiator. Its radiation spectrum is only a function of wavelength and its temperature. A small hole in a large box painted black inside has the characteristics of a blackbody. The multiple reflections inside the box allow an equilibrium to become established between emission and absorption. An object covered in lampblack (fine soot) is a good approximation in the visible and IR regions of the spectrum.



(4d)

Wien's Displacement Law

The Planck radiation curves above show the power output of a blackbody at various temperatures as a function of wavelength. At very high temperatures the peak of the emission is well into the X-ray part of the spectrum whereas at very low temperatures the only significant emission is in the microwave region. The relationship of the peak of the spectrum to the temperature of the body is given by Wien's Displacement Law:

$$\lambda_{max}T = \text{constant}$$

(2.9×10^{-3} metre kelvin for the curves shown)
(rather confusingly written m K which could be misinterpreted as millikelvin) .

hence

$$\lambda_{max} \propto \frac{1}{T}$$ where T must be in kelvin.

Switch on an electric fire or cooking ring. As it warms up, first feel the warmth but note that it does not glow visibly (the peak of emission in IR). After a while, watch it glow dull red and finally orange as its peak of emission moves into the visible.

A photographic flash gun produces a rather white light. Use the curves to predict the temperature of the inside of the flash tube. Compare this value with the colour temperature printed on the outside of film boxes. Why does daylight film produce red/orange results when used indoors? What is the temperature of a normal light bulb?

The colour of the Sun can be understood by examining the curve for 5 700 K. An object at about this temperature would radiate fairly evenly over the visible region and so look white. The surface (photosphere) of the Sun approximates well to this curve.

The primordial fireball at the beginning of the universe has expanded and cooled over about 1.5×10^{10} years. Radio astronomy observations of the microwave background as a function of wavelength give a temperature of about 2.7 K for these remains of the Big Bang. The slight fluctuations in this radiation are important in explaining the formation of the galaxies. If the radiation had been very smooth, it would have been hard to explain how gravity could have pulled together the hydrogen and helium. Fortunately, the COBE satellite found sufficient variation to support the Hot Big Bang theory.

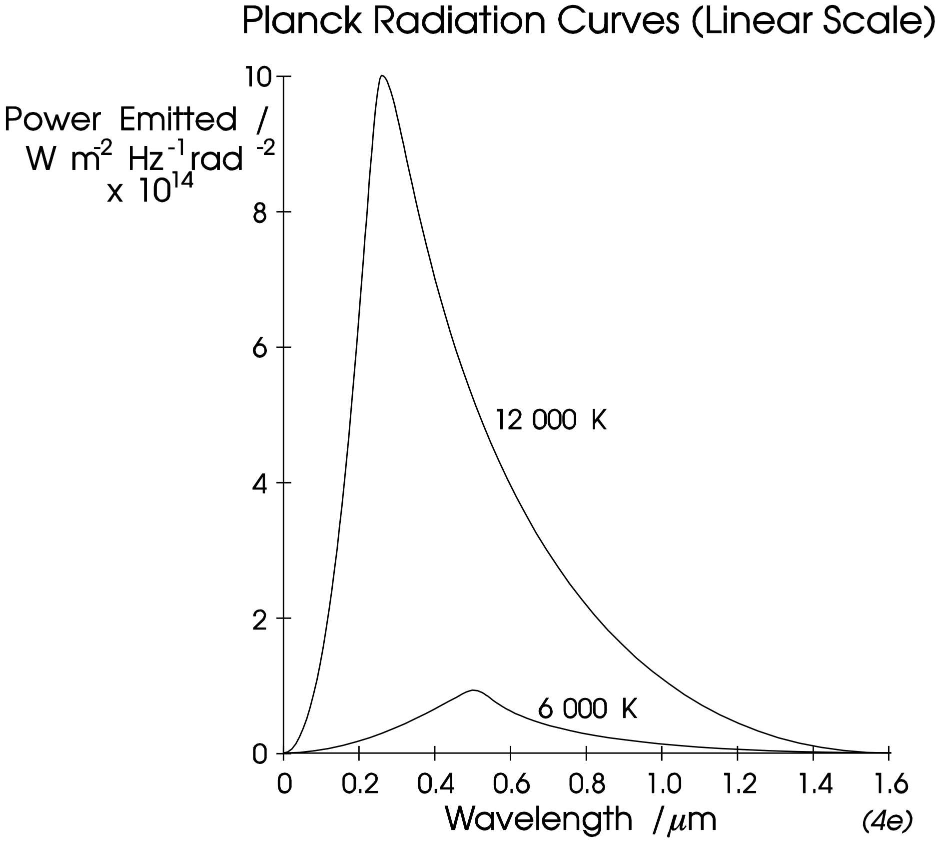
Stefan's Law

If the total energy output of a blackbody is found by integrating the Planck curve and this result is then integrated to cover one hemisphere the result is Stefan's Law (Stefan-Boltzmann Relation):

$$B = \varepsilon \sigma T^4$$

where B is the total brightness - the total power emitted per unit area of the source in $\mathrm{W\,m^{-2}}$,
ε is the emissivity; 1 for a blackbody and < 1 for real surfaces,
σ is Stefan's Constant $= 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$ and
T is the absolute temperature in kelvin.

The power output per square metre of a star is very strongly related to its temperature! As an example consider two black bodies, one at 6 000 K and one at 12 000 K. The graph shows the results of measuring their brightness experimentally:



This graph is a good example of the quality of the links between theoretical and experimental physics. The area under the 12 000 K curve is 16 times the area under the 6 000 K curve; $(12\,000/6\,000)^4$ as predicted by Stefan's Law. The peak of the 12 000 K curve is at 0.25 μm whereas the peak of the 6 000 K curve is at 0.5 μm as predicted by Wien's Law using $\lambda_{max} \propto 1/T$.

These curves are very useful in estimating the temperature of a distant object since the *peak* is not seriously affected by absorption of the radiation in the interstellar medium or the atmosphere or by the distance estimate of the star.

Good example stars are: Spica - bluish – 20 000 K
Betelgeuse - orange/red – 3 000 K.

Classification of Stars

Principles of the use of Stellar Spectral Classes

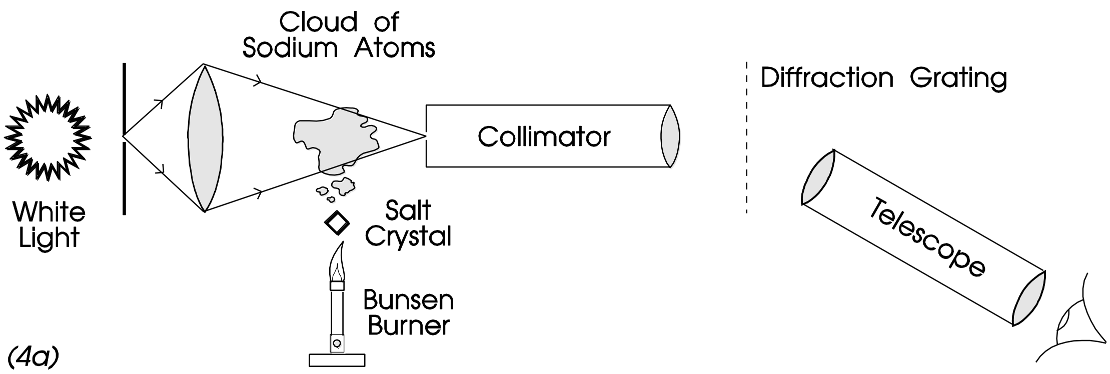
Spectra

An atom may gain energy by collision with a particle or by absorbing a photon (quantum). The atom stores this energy (a few electron-volts typically) by allowing an electron to be raised to an energy level that is higher than its normal unexcited level. The electron usually falls back to its original level after a very short time (about 10^{-9} s) and in doing so emits a photon of energy. As this transition is between two precisely defined levels, the energy (ΔE) of the photon is precisely defined and hence so is its frequency (f) and wavelength (λ). The equations are:

$\Delta E = hf$ and $c = f\lambda$ where h is Planck's Constant
and c is the velocity of light.

These precisely defined levels only occur in isolated atoms that are themselves found only in a gas. Hence if a heated gas is observed with a spectroscope, line spectra are seen. By way of contrast, in solids or liquids the narrow levels that give rise to the lines are broadened by interactions between adjacent atoms. Ranges of permissible energies result and band spectra are seen.

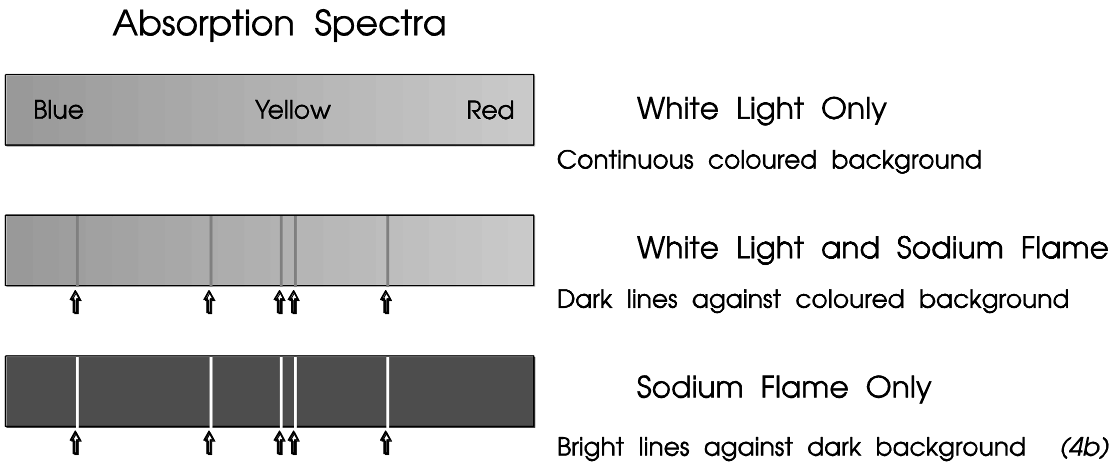
Experiment to Observe Absorption Spectra



(4a)

The apparatus in the diagram can be set up in the laboratory. If white light is observed through the spectrometer then a continuous spectrum of colour from red to blue is seen. When passed through a cloud of sodium atoms, particular wavelengths corresponding to energy level differences in the atoms are absorbed. When reradiated, instead of the light being directed only towards the collimator, it is spread out in all directions. Hence the collimator receives less of these specific colours.

The following graphic shows what would be seen in the telescope on one side of the centre line.

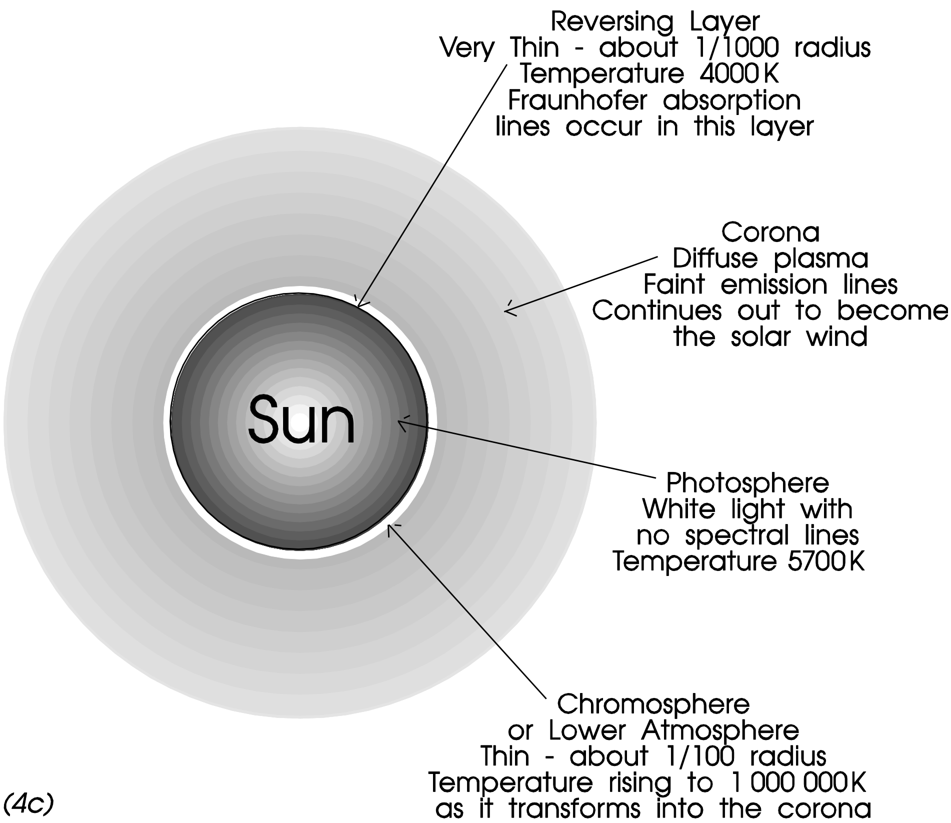


(4b)

The first spectrum is the continuum spectrum from the surface of a hot object, in this case the filament of the lamp. The dark lines in the second spectrum are the absorption lines of isolated sodium atoms in the cloud. These lines correspond exactly to the emission lines in the third spectrum where the white light is turned off.

Solar Spectrum

The same process in the hot gaseous atmosphere surrounding the Sun (or any star) gives rise to absorption and emission lines. The absorption lines were first reported by Fraunhofer in 1814. He saw them in sunlight, in moonlight and in the light from Venus. He correctly concluded that the Moon and Venus shone by reflecting sunlight.



(4c)

White light with no lines is emitted from the visible disc (photosphere) of the Sun. As it passes through the slightly cooler 'reversing layer', absorption occurs. This layer is so called because the temperature at first falls and then rises again with increasing distance from the surface. These absorption lines can be seen with a hand-held spectroscope pointed at the sky, even on a cloudy day.

The Sun's chromosphere can be seen at times of total eclipse as a thin pinkish band on the rim of the Sun. Both the chromosphere and corona are tenuous and have little effect except to add faint emission lines. These lines are only easily detected at times of total eclipse.

The spectrum observed by Fraunhofer contained new lines not seen during experiments on terrestrial materials. The lines enabled helium to be identified in the Sun and subsequently on the Earth.

Try viewing the Fraunhofer Lines using a spectrometer. Set up a small mirror near the window of the laboratory to catch some direct sunlight. Arrange the mirror to be at exactly the same height as the slit on the collimator. Direct the light into the slit; you will need to continually turn the mirror to cancel out the rotation of the Earth! Cover the spectrometer with a dark cloth to cut out the stray light. Using a 300 line per millimetre grating you will see hundreds of dark lines. Look for a prominent pair of lines in the yellow part of the spectrum. Without adjusting the spectrometer, shine a sodium lamp into the slit. The dark lines will almost certainly correspond to the bright D-lines of sodium, so you have proved that there are sodium atoms between you and the surface of the Sun!!

The lines from isolated atoms and ions and the bands from isolated molecules extend from the UV through the visible, the IR and into the radio spectrum. They are important in many types of measurement.

Spectral Classes

By careful observation of the spectra of many stars it is possible to identify a number of distinct classes. These were originally in alphabetical order of the complexity of their spectra and in order of decreasing temperature. However as the understanding of stars improved the classifications were modified, so the present list is:

Oh, Be A Fine Girl, Kiss Me Right Now, Sweetheart!

The mnemonic is attributed to the American astronomer Henry Norris Russell of H-R Diagram fame. The classes 'R and N' have been merged in class 'C' since Russell's time. After class 'M' the steady decrease in temperature down the table is lost.

Class	Temperature	Colour	**Principal Spectral Characteristics** (Continuum from the photosphere, absorption lines from the shell of gas.)	Examples
O	> 25 000 K	Blue	*Ionised helium (He II) lines.* Highly ionised species C III, N III, O III and Si IV. Strong UV continuum.	10 Lacertae
B	25 000 K to 11 000 K	Blue	Neutral helium (He I) lines. *Hydrogen (Balmer n = 2) lines visible.* Lower ions C II, O II and Si III.	Rigel Spica
A	11 000 K to 7500 K	Blue	*Strong hydrogen lines.* Mg II and Si II strong. Ca II weak.	Sirius Vega
F	7 500 K to 6 000 K	Blue -White	Hydrogen lines weaker, Ca II stronger. *Lines of neutral and singly ionised metals.*	Procyon
G	6 000 K to 5 000 K	Yellow - White	*CaII very strong. Neutral metal atoms strong, ions weaker.*	Sun Capella
K	5 000K to 3 500 K	Orange	Neutral metals predominate. *Molecular bands (CH, CN) visible.* Blue continuum weak.	Arcturus Aldebaran
M	< 3 500 K	Red	*Strong molecular bands, particularly TiO.* Some neutral lines. Red continuum strong.	Betelgeuse Antares

The items in *italics* are the key features in the spectra. The abbreviation Si III indicates the *double* not triple ionisation of the silicon atom. Si I is the neutral state of the atom.

The main trend disclosed by the table is that:

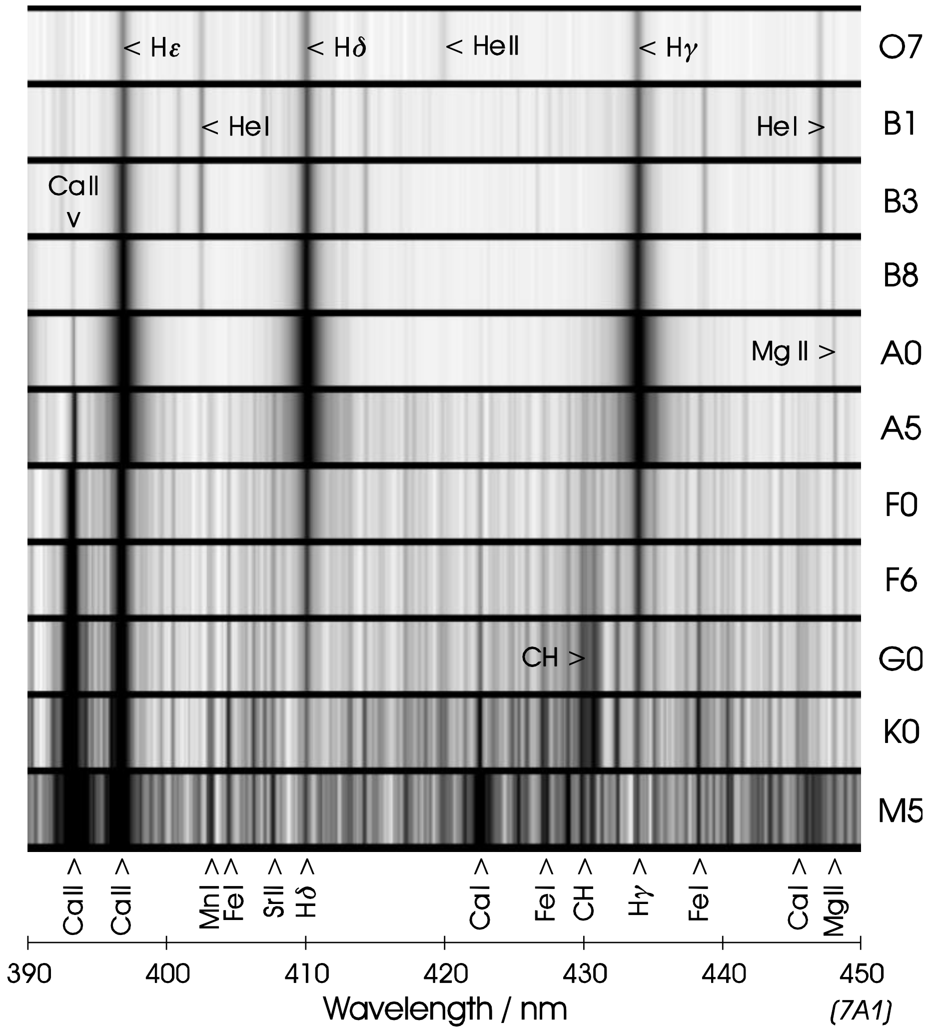
- High temperatures – atoms are highly ionised and much ultra-violet light is emitted
- Medium temperatures – ionisation is usually single and the continuum peak moves into the visible
- Low temperatures – molecules are able to form and the continuum peak moves into the red.

Extension Material

Class	Temperature	Colour	**Principal Spectral Characteristics** (Continuum from the photosphere, absorption lines from the shell of gas.)	Examples
C	5 000 K to 3 500 K		Carbon stars. Strong bands of C_2, CN and CO. TiO absent.	19 Piscium
S	< 3 500 K	Red	Similar to M stars but with Zirconium and Lanthanum monoxides.	S Ursa Majoris
W	> 25 000 K	Blue	O stars with bright *emission* lines from rapidly expanding shell gas.	Wolf - Rayet Stars

End of Extension Material

Spectra of Main Sequence Stars



(7A1)

In this set of spectra (kindly provided by Steve Fossey of University College Observatory, London) singly ionised helium (He II) is only seen in the hot O star. Neutral helium (He I) is seen clearly in the B stars. At lower temperatures there is no sign of helium at all.

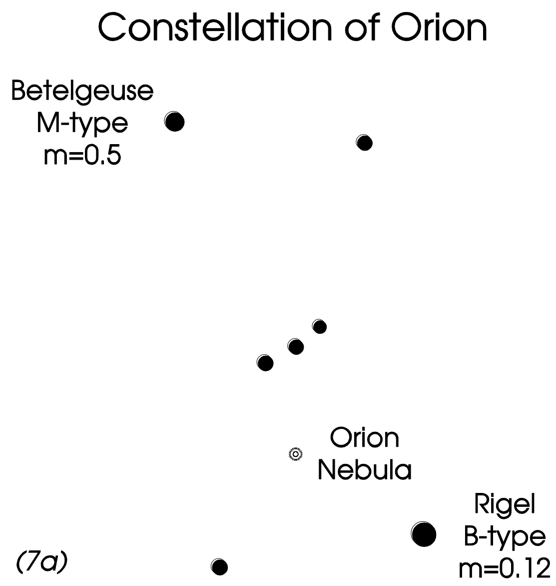
In the O, B and A stars the three hydrogen lines are particularly strong. The temperature is sufficiently high to hold hydrogen atoms in the $n = 2$ state before excitation to produce these Balmer lines.

The single ionised calcium (Ca II) lines appear increasing strong in the cooler F, G, K and M stars. Ca I is visible in the K and M spectra.

The molecule CH is visible in the G spectrum and strengthens in the cooler K and M stars.

In the M star, metal lines such as Mn I, Fe I, Sr II, Ca I, Mg II are strong. Temperatures of less than 3 500 K are low enough to allow neutral metal atoms to exist.

The constellation of Orion has two good example stars. Betelgeuse is an M-type Red Giant (and is slightly variable) and Rigel is a B-type bright blue star. Their apparent magnitudes (m) are very similar but Betelgeuse is noticeably orange even to the naked eye. The Nebula in Orion consists of stars forming in a dense cloud of gas and dust. The stars illuminate the cloud and make an impressive sight in even a small telescope. This constellation is visible in the Autumn, Winter and Spring. You should make an effort to observe it even if only with the naked eye.



Extension Material

Population 1 and 2 Stars

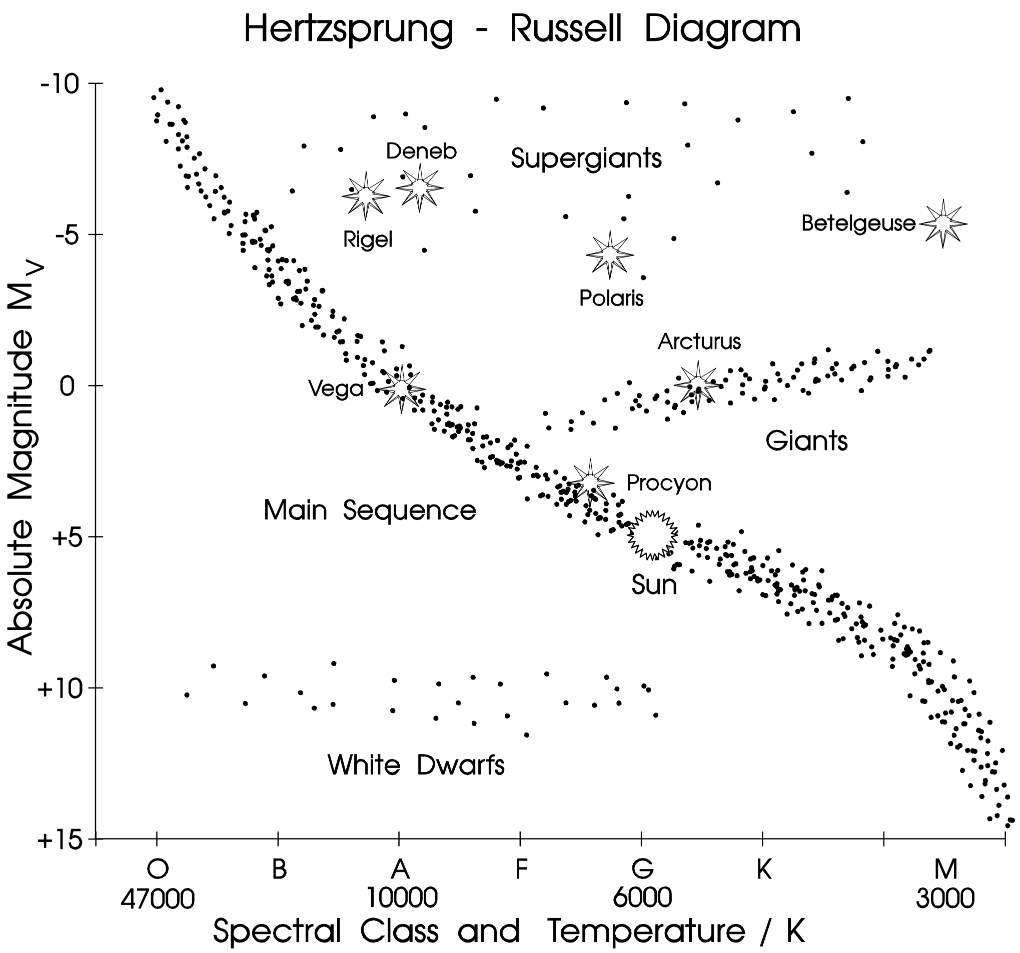
Population I stars are relatively young and are found in the spiral arms of the Galaxy. They contain relatively large proportions of heavier elements indicating that they formed from materials that had undergone nuclear reactions in earlier generations of stars.

Population II stars are relatively old and are found in the spherical halo of the Galaxy. They contain relatively small proportions of the elements heavier than helium. This implies that they formed at an early stage in the lifetime of the Galaxy, before there was time for the heavier elements to be synthesised. They have high velocities relative to the Sun and the Galactic plane. Globular Clusters are composed of population II stars.

End of Extension Material

The Hertzsprung – Russell Diagram

When a large representative sample of stars in the Galaxy is plotted as shown below, certain trends become clearly visible.



(7b1)

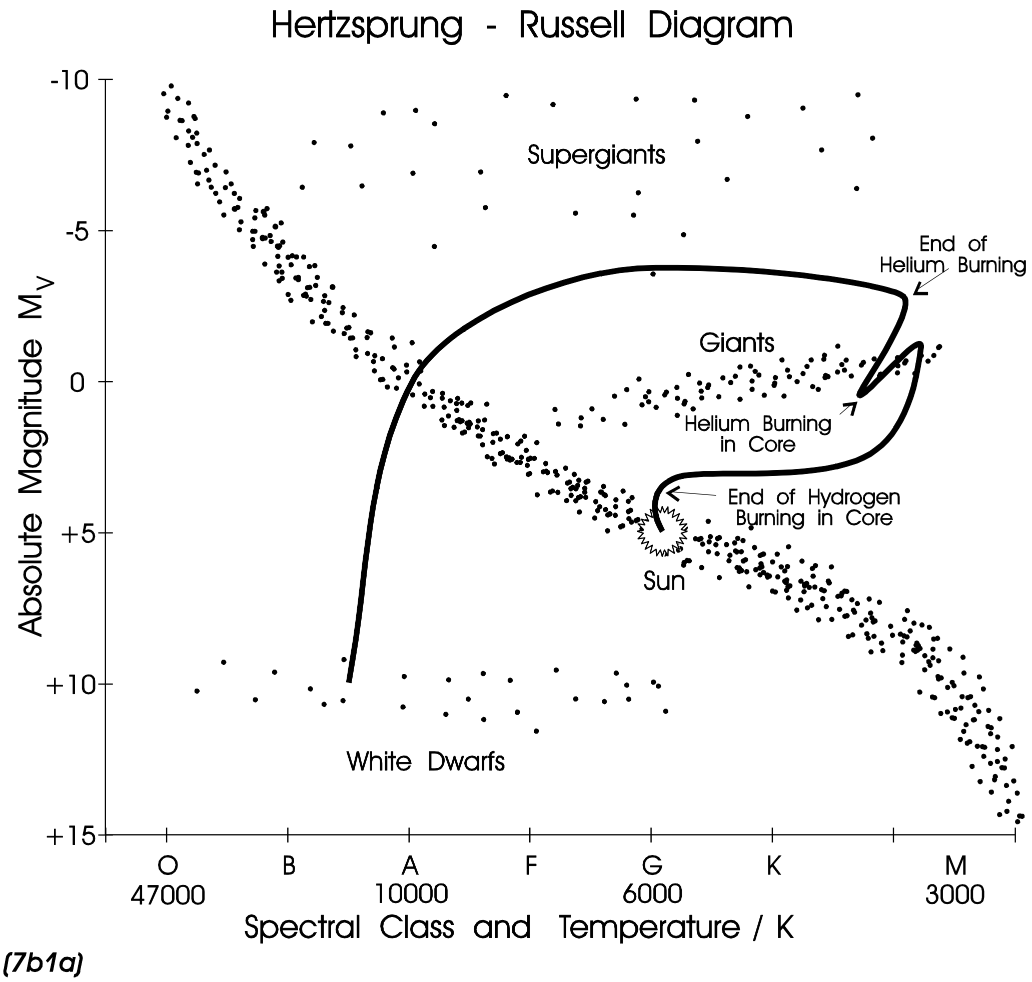
Main Sequence Stars

A star is born when a cloud of cold gas and dust slowly collapses under the force of its own gravitational attraction. The cloud must be cold in the early stages of formation otherwise the motions of the atoms and molecules will allow them to escape from the gravitational forces. It is unlikely that the cloud will be symmetrical or stationary as it starts to collapse. This will give rise to rotation in the newly forming star and quite possibly leave a disc of material that will go on to form planets.

As the collapse continues the gravitational potential energy of the cloud is converted into heat at its core. If the cloud is too small it will only form a Brown Dwarf similar to Jupiter. (In the motion picture 2010, the unspecified force throws massive tablets of stone into Jupiter to increase its mass and making it form a second Sun in our Solar System!!). If the cloud is about one solar mass it will heat up its centre sufficiently to allow the nuclear fusion of hydrogen to helium to occur – a temperature of about 1.5×10^7 K is required.

Stars on the main sequence that have formed recently are composed of about 73% hydrogen, 25% helium and 2% of heavier elements, measured by mass. It is interesting to compare these ratios with the values predicted for just after the Big Bang – 75% hydrogen, 25% helium and a trace of lithium. The reduction in hydrogen and the growth of the heavier elements is assumed to be the result of fusion in the cores of massive stars. Perhaps surprisingly, the percentage of helium in the free gas of the Universe remains fairly constant, considering that all stars convert hydrogen to helium in their cores. However, stars that explode as supernovae convert most of this helium into heavier elements. Stars that do not explode keep most of the helium locked up in their cores.

Stars spend most of their life on the main sequence and evolve off it when they finish their hydrogen burning phase. Stars up to about 4 solar masses are able to burn hydrogen to helium and perhaps some helium to carbon. Stars of greater mass and hence with high internal temperatures are able to synthesise elements up to iron in their cores.



(7b1a)

Extension Material

The Energy Source in the Sun

The Sun creates 91% of its energy in the PP I chain. The initial protons are combined to produce one helium nucleus, a positron, an electron neutrino, a gamma ray and two protons.

$$p^+ + p^+ \rightarrow {}^2_1H + e^+ + \nu_e$$
$${}^2_1H + p^+ \rightarrow {}^3_2He + \gamma$$
$${}^3_2He + {}^3_2He \rightarrow {}^4_2He + p^+ + p^+$$

where e^+ is a positron and ν_e is an electron neutrino. The probability of two protons encountering each other with sufficient speed to start the reaction is surprisingly low. In the Sun the average time is 14 billion years. However there are many protons, so this is still the main energy production route.

When the positrons have annihilated and all other energy is accounted for, the reaction releases 26.8 MeV (4.29×10^{-12} J) which is millions of times more energy than a chemical reaction.

Stars of high mass spend a much shorter time on the main sequence than do low mass stars. With about one solar mass a star would spend nearly 1.0×10^{10} years before evolving off it whereas with 15 solar masses it would spend as little as 1.0×10^7 years on it – a thousand times shorter.

The Age of the Sun

One justification for taking the age of the Sun to be 1.0×10^{10} years comes from looking at the geology of the Earth and the Moon and of the composition of meteorites and comets. Using the half-life of isotopes of heavy elements it is possible to estimate the age of the oldest rocks to be 4.55×10^9 years. The solar system must have formed before this time. Another check is the rate at which the Sun is emitting energy. Assuming that nuclear fusion is the energy source it is possible to use the famous Einstein equation $E = mc^2$ to calculate the rate at which mass is consumed. *Look up the solar luminosity and work out the rate of loss of mass. Use the mass of the Sun to calculate what percentage of its mass has been burnt, assuming an age of 1.0×10^{10} years.. You should find that it has only used a small fraction of its mass!*

An interesting exercise is to work out the length of time the Sun would shine if it were generating its energy by chemical reactions. Take the well-known energetic reaction of exploding hydrogen with oxygen to form water. Each mole of water produced has a mass of 18 grams and yields about 200 kJ of energy. Use the mass of the Sun and its *luminosity to calculate the time to run out of fuel. You should find that we would not be here to do the calculation!!*

The Core of the Sun

It is possible to estimate the temperature of the core of the Sun as follows. If protons are to come close enough to combine in nuclear fusion, they must overcome the electrostatic force of repulsion that arises from their positive charges. This means that they must have a very high speed, which implies a very high temperature. Equating the kinetic energy of a proton with the height of the potential mountain it must climb gives an estimate of the temperature.

The kinetic energy (E) of a simple particle in a gas is given by:

$$E = \frac{3}{2}kT$$

where k is Boltzmann's Constant and

T is the absolute temperature in kelvin.

The potential energy (E) required for two charges to approach each other is given by:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{QQ}{r}$$

where Q is the charge on the proton in C,

r is the separation in m and

ε_0 is the Permittivity of a vacuum.

Try equating these expressions using 1.6×10^{-19} C as the charge and 1.5×10^{7} K as the temperature. Does the separation compare well with the size of the nucleus? Is it much hotter than the surface of the Sun?

The energy released in the core has to make its way to the surface. Conduction plays little part in the transfer as the density of material in the outer layers is too low.

In our Sun the fusion core extends out to about one quarter of the way to the surface, beyond this there is a plasma region out to about 85% of the way to the surface. Radiation is the main means of transport in these regions but it is not easy for photons to escape. The distance a photon can travel in the core before it scatters off a particle is only a fraction of a millimetre! The mean free path only increases to a few centimetres in the outer regions. The whole journey from the centre takes a photon about 10 million years.

In the outer 15% of the journey, convection is the main transport mechanism for the energy. In these outer layers the density is comparable with that of the air on Earth. With the strong heating from below, convection currents form, carrying the energy outwards.

The layers of the Sun are supported against gravitational collapse by pressure from within. The collisions of the particles give rise to conventional gas pressure but the layers are also supported by the pressure of the photons scattering off the particles - radiation pressure! In stars of about 5 solar masses the gas and radiation pressure are equal. In smaller stars like our Sun, the gas pressure is the larger, but not by much. In 100 solar mass stars the radiation pressure dominates. As a star runs out of fuel these sources of pressure reduce and the star collapses.

End of Extension Material

Classification of Stars

White Dwarfs

These are the cores of medium sized stars that were originally similar to our Sun. They have lost much of their outer material and are now significantly smaller than the Sun (perhaps 0.02 of the radius). They are at the end of their fusion lives and are using residual heat and gravitational potential energy to shine. Eventually they cool to black dwarfs that might constitute a significant proportion of the mass of the Universe! White dwarfs have densities between 2.5×10^7 and $6 \times 10^{11}\,\mathrm{kg\,m^{-3}}$. These very high values indicate that the material is so compressed that the electron shells have collapsed and the nuclei are almost touching each other - electron degenerate material. They *must* contain *less* than 1.4 solar masses of material (the Chandrasekhar Limit) otherwise they would collapse into neutron stars.

$M_v \approx +10$ Classifications O, B or A.

Brown Dwarfs

These are low mass, low luminosity main sequence stars. They have only just enough mass to raise their core temperatures sufficiently to start nuclear fusion. Stars of less than 0.1 solar masses never start fusion reactions and shine only by the release of gravitational potential energy. N.B. Not all red stars are giants and not all dwarf stars are white!!

Giants

These are stars of at least one solar mass that are approaching the end of their hydrogen burning phase and have an expanded outer gaseous shell. The core collapses a little and its temperature rises allowing the helium burning that produces heavier elements to begin. Hydrogen burning continues in the middle layers of the star.

Eventually the outer layers of gas will be driven off by the increasing temperature. The core will go on to become a white dwarf. This process takes hundreds of thousands of years and we can see it in the form of 'Planetary Nebulae'. These are beautiful shell-like structures of gas that have been pushed away from the ageing star.

$M_v \approx 0$ Classification B, A, F, G or K.

Supergiants

These are massive stars, typically 8 to 100 solar masses. The very high core temperatures (1×10^8 K) cause the fuel to burn at a prodigious rate. The outer layers have expanded to many times the radius of our Sun. Strong winds develop, ejecting material from the star - particularly noticeable in carbon stars. Once the central hydrogen is used up, helium is fused into carbon and so on up to iron.

Beyond this point in the periodic table the reactions require the input of energy. The core then suffers a great collapse and on rebounding produces a massive supernova explosion, blowing off the outer layers of gas and ending the short life of the star. A neutron star may be left at the centre of the explosion if the mass remaining is greater than 1.4 solar masses. In the last few seconds of the collapse, the neutron flux in the core is very high and iron nuclei absorb neutrons to create the heavy elements found in the rest of the galaxy. To obtain the proportions of heavy elements found in our region of the galaxy it is likely that the material has been through the supernova process about five times!

$M_v \approx -10$ Classification B, A, F, G, K or M.

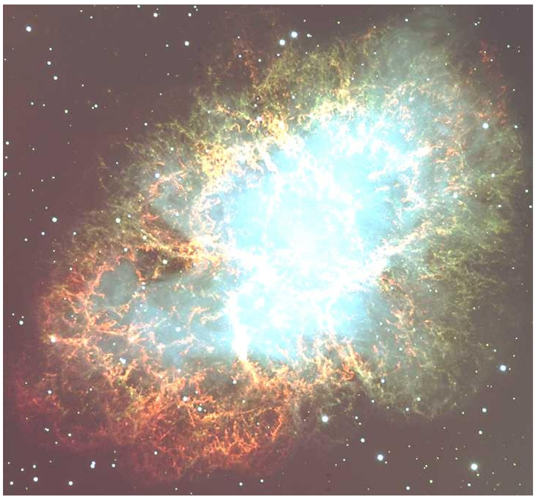
Classification of Stars

Supernovae, Neutron Stars and Black Holes

The size of a star is controlled by the balance between the gravitational attraction pulling inwards and thermal forces pushing outwards. The outward forces are made up of pressure from the hot material and the pressure generated by photons bouncing (scattering) off the particles of material.

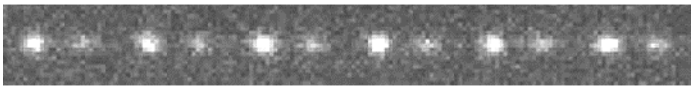
A star of similar mass to that of our Sun will evolve from this equilibrium and enter the Red Giant phase after burning the hydrogen and helium in its core.

However a star greater than about four solar masses will burn most of its hydrogen then collapse a little and raise its core temperature. It will then start to burn helium in its core. It develops an onion-like structure with the inner layers having higher and higher temperatures. In the innermost layers, elements up to iron are created. Synthesising elements beyond iron requires energy input rather than being a source of energy. Without further sources of nuclear energy the core starts to cool and the gravitational forces overcome the thermal forces causing the star to collapse very rapidly. The resulting compression may well produce a central neutron star. The rebound throws off a large proportion of the star's outer material. Initially the colour of the star is the red of the giant's gas envelope but as the ejected material races away, the shock wave heats the debris and the star becomes quite blue. The explosion results in a billion-fold increase in the brightness of the star, which lasts for several weeks. This type of supernova event creates objects such as the Crab Nebula. The image was taken using ESO's Very Large Telescope.



If the core is larger than 1.4 solar masses, as in the Crab, a spinning neutron star is left behind. It is surrounded by a large expanding cloud of debris. The neutron star has the radius of a large city centre and rotates about 30 times a second. This rotation produces pulses of energy, rather like a lighthouse, that are detectable on Earth as a Pulsar. The requirement for a pulsar to form is that the magnetic field is not aligned with the spin axis of the neutron star. The rotation then causes major disturbances in the ionised debris, heating it strongly and causing synchrotron radiation emissions.

The image below shows the optical pulses emitted by the Crab Pulsar (ESO).



If the original star in the Crab had been a little more massive, the core would have exceeded the 2.5 solar mass limit and would have collapsed into a Black Hole. In that state the escape velocity from the core would exceed the velocity of light and no signals would be able to escape.

From Newton's Laws of Gravitation it can be shown that the energy required to escape from an object of mass (M) when at a distance (r) from its centre is given by:

$$Energy = \frac{GMm}{r}$$ where G is the Gravitational Constant.

If this energy is only available as a single pulse, as in the case of a bullet travelling with an initial velocity (v), the kinetic energy can be equated to the formula above:

$$\therefore \frac{1}{2}mv^2 = \frac{GMm}{r}$$

This equation should contain relativistic corrections on each side but fortunately they cancel out!

$$\therefore v = \sqrt{\frac{2GM}{r}}$$

When this velocity equals the velocity of light (c) the formula becomes:

$$\therefore c = \sqrt{\frac{2GM}{r}}$$

Rearranging:

$$\therefore r = \frac{2GM}{c^2}$$

This defines the Schwarzschild Radius (r). Any object closer than this distance would require a velocity greater than that of light to escape – an impossibility. Hence the Schwarzschild Radius defines a sphere that is the Event Horizon of the Black Hole. Inside this sphere nothing is visible and nothing escapes.

Can you calculate the radius of a neutron star of one solar mass for its radius to be equal to its Schwarzschild Radius? Take the density of neutron material to be 1×10^{17} *kg* m^{-3}. *Try the calculation again using the density of water.*

In fact Black Holes can be expected to be very bright in some circumstances since they will attract nearby material. This material will undergo very energetic collisions as it falls inwards and so will emit visible light, UV and particularly X-rays. A most interesting class of objects recently discovered is the Symbiotic Binary system. Two stars in a binary system evolve naturally until one passes through its red giant phase and undergoes a supernova explosion to become a black hole. The companion star is not destroyed by the explosion and sometime later goes on to its own red giant phase. As it expands, material from its gas shell is pulled down into the black hole. The resulting compression heats the gas so strongly that it only emits most of its energy in the X-ray region of the spectrum! These objects would have been hard to find without detectors mounted in satellites since the Earth's atmosphere absorbs X-rays completely!!

Type Ia Supernovae

A Type Ia event occurs when a white dwarf, composed mainly of carbon and oxygen, is one partner in a binary star system. As the other star goes into its red giant phase, gas is drawn down onto the surface of the white dwarf. Some of this material burns to helium and then carbon and oxygen. When the dwarf passes the Chandrasekhar limit of 1.4 solar masses, the carbon core cannot resist the gravitational forces. The temperature rises, the core ignites and the star explodes. It is also possible for a solitary star of a few solar masses to evolve into helium star with a carbon and oxygen core. This explodes in the same way.

In either case the resulting explosion burns about one solar mass of carbon and oxygen. The product is nickel 56, which undergoes radioactive decay to cobalt 56 and finally to iron 56. This decay chain explains the exponential decay (60 day half-life) of the brightness of the Type Ia supernovae. This characteristic behaviour allows Type Ia supernovae to be identified unambiguously. The standard methods for comparing magnitudes can then be used to compare their distances.

The advantage of using Type 1a supernovae as distance indicators is their extremely high luminosity that allows them to be observed in distant galaxies. The explosion dramatically increases the surface area and temperature of the star causing it to brighten by a factor of a billion or more. The peak absolute magnitude is expected to be similar for all stars of a given mass. This allows the Type 1a supernovae to be used as Standard Candles to measure distances across the Universe.

Extension Material

Type Ia Supernovae Distance Calculation

Using the formula:

$$2.512^{m-M} = \frac{I_a}{I_b}$$

where I_a is the intensity *after* the explosion and I_b is the intensity *before* the explosion.

$$\therefore 2.512^{m-M} = 10^9 \qquad \text{then taking logs base 10}$$

$$\therefore \log 2.512^{m-M} = \log 10^9$$

$$\therefore (m-M) \times \log 2.512 = 9$$

$$\therefore m - M = 22.5$$

Thus the star increases in brightness by 22.5 magnitudes. This is equivalent to a bright star in the night sky suddenly approaching the brightness of the Sun!

Using the inverse square law, the increased distance that a supernova can be observed may be estimated:

$$\therefore \frac{d_a}{d_b} = \sqrt{10^9} = 3.2 \times 10^4$$

where d_a is the distance of visibility *after* the explosion and d_b is the distance of visibility *before* the explosion.

The distance is increased by a factor of 30 000 times over the distance that the unexploded star can be seen. They can be detected out to distances of about 1000 Mpc.

End of Extension Material

Cosmology

Doppler Effect

If an object emits electromagnetic waves and it is moving relative to an observer, then the received frequency and wavelength of the waves are affected by the relative motion. If they approach one another, the wavelength appears shortened (blue shift) and if they recede from one another it appears lengthened (red shift). If the source is emitting a known wavelength then the relative velocity may be calculated using:

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$ where $\Delta\lambda$ is the change in wavelength,

λ is the original wavelength in m,
v is the relative velocity in $\mathrm{m\,s^{-1}}$ and
c is the speed of light in $\mathrm{m\,s^{-1}}$.

The relative velocity v is taken as positive when the source and observer are approaching each other. The change in wavelength $\Delta\lambda$ will then be negative, implying an decrease in wavelength – a blue shift.

Doppler Calculations

An object receding rapidly can cause a line normally in the UV part of the spectrum to shift into the visible range. The hydrogen line in the Lyman series, Lyman α, is emitted at 121.6 nm that is well into the UV. If this line is actually observed at 530 nm then using:

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$ and substitution gives

$$v = \frac{(530 - 121.6)}{121.6} \times 3 \times 10^{8} = 1.01 \times 10^{9}\,\mathrm{ms^{-1}}$$

This is clearly impossible as v must not be greater than c!! In fact the formula only works for modest velocities where $v < 0.1$c, i.e. $z < 0.1$. At $z = 0.2$ an error of 12% is introduced.

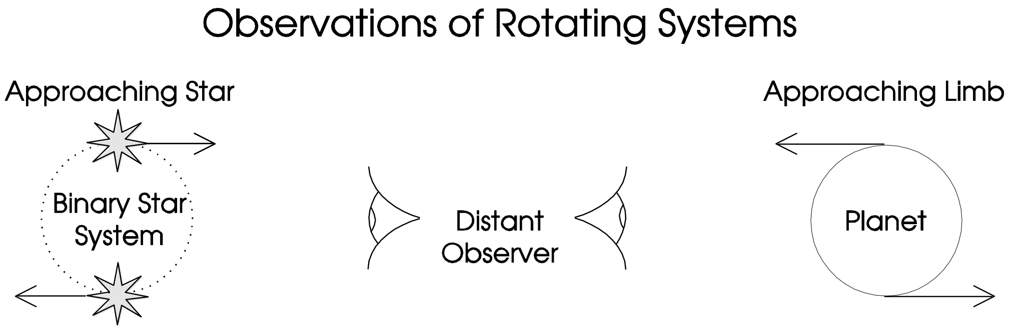
A better approximation can be made using the Special Relativity version of the formula:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}} - 1$$

Use this relativistic formula to show that the value of v is $2.7 \times 10^{8}\,\mathrm{m\,s^{-1}}$ that is 90% of the speed of light and corresponds to $z = 3.36$. This formula is part of a rather superficial theory about the expansion of the Universe, the full depth of which can only be explored through the General Theory of Relativity.

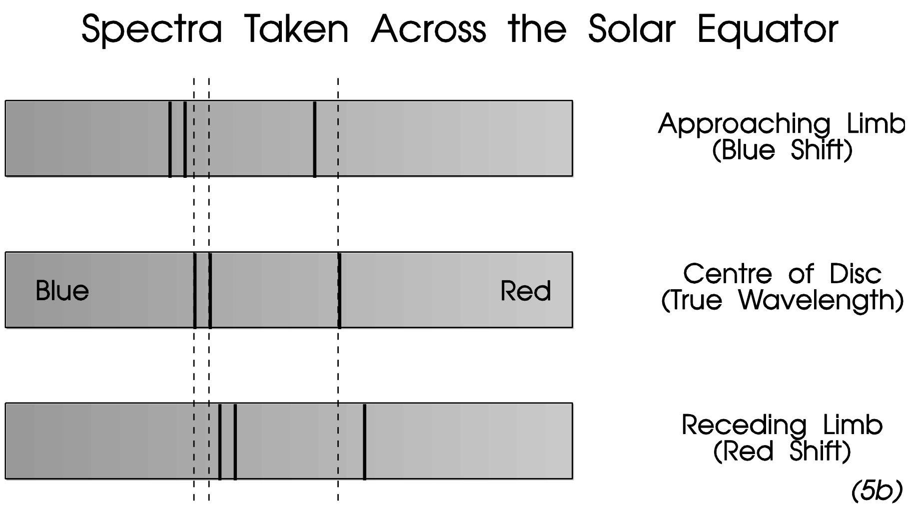
Rotation

When a rotating star, binary system, planet or set of planetary rings is viewed from a distance, one component (limb or edge) appears to be receding and the other approaching.



(5a1)

The Doppler Shift observed in this way gives the tangential velocity (v) of the limb. Using $v = r\omega$ and $\omega = 2\pi/T$ the period of rotation (T) may then be calculated. This assumes that spectral lines are observable in the system. A reference line must also be taken from the system. In the case of a star or binary system the mean wavelength may be used. In the case of a planet or ring, the reference is the wavelength of that part that is moving at right angles to the line of sight; i.e. has no velocity in the direction of the observer.



(5b)

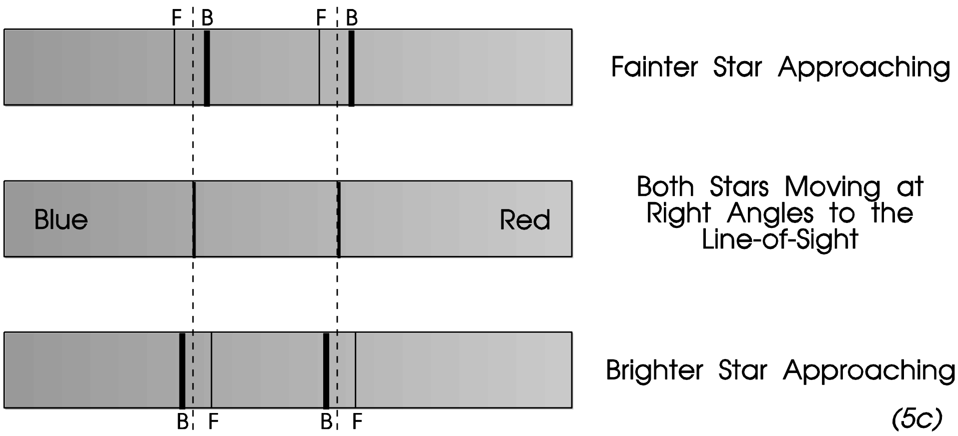
For all stars except the Sun, it is not possible to focus on each limb independently. The result is that we see a set of broadened lines whose widths can be used in a similar way to that described in the previous section.

Spectroscopic Binary Systems

Binary systems are often so distant and their components so close together that they cannot be resolved as two separate objects in a telescope. The only way to measure their orbits is by spectroscopy; hence such objects are called Spectroscopic Binaries.

Some Spectroscopic Binary systems show two distinct sets of lines, one set from each star. As the stars orbit each other so the lines move from longer to shorter wavelength and back again. This allows the period of the system to be measured. From the Doppler Shifts, the velocities of the stars may also be calculated. The period and velocity can be combined to obtain the distance between the stars if the inclination of the orbits can be estimated. It can be shown that the ratio of the orbital radii is equal to the ratio of the masses. This is a powerful measuring technique considering the stars are too far away to be seen individually. It is most useful if the stars are eclipsing binaries as their orbital plane must be edge-on to the observer for the eclipse to be visible. This makes the calculations straightforward.

Spectra of a Binary Star System



(5c)

Extension Material

Binary Star Orbits

The orbital theory for two stars of equal mass is quite simple. They will orbit their common centre of mass that will be half way between them and be held in their orbits by their mutual gravitational attraction.

The centripetal force (F) required to maintain an orbit is given by:

$F = mR\omega^2$ where m is the mass of the star in kg,
R is radius of the orbit in m and
ω is the angular velocity in radian s^{-1}.

The gravitational attraction (F) between two masses is given by:

$$F = \frac{Gmm}{r^2}$$ where G is the Gravitational Constant,

m is the mass of one star in kg and
r is the distance between the stars in m.

Now for this symmetrical binary system R is half the size of r, as each star is a distance R from the centre of mass. Hence:

$$m\frac{r}{2}\omega^2 = \frac{Gmm}{r^2}$$ and by definition

$$\omega = \frac{2\pi}{T}$$ where T is the period of each orbit in s.

Substituting and rearranging:

$$m = \frac{2\pi^2 r^3}{GT^2}$$

if the masses are equal then M may be defined as twice m. Hence:

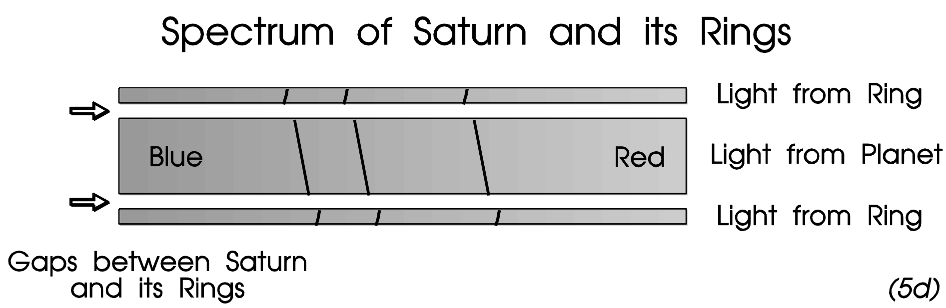
$$M = 2m = \frac{4\pi^2 r^3}{GT^2}$$ where M is the total mass of the system.

The formula is defined in this way in the London Modular syllabus.

Objects Observed in Reflected Sunlight

The key point to remember here is that the Doppler Effect occurs twice. The particle or planet is moving relative to the incoming sunlight and then the object re-emits the light whilst moving relative to the observer. Therefore Fraunhofer lines in the sunlight suffer twice the normal Doppler Shift, hence $\Delta\lambda/\lambda = -2v/c$ if the Sun, Earth and particle are in a straight line. This theory applies to studies of rotation of the planets and their rings and the velocities of asteroids.

A typical spectrum of Saturn and its Rings is show below:



The slant of the absorption lines has been exaggerated and only a few are shown. These lines are Fraunhofer lines from the Sun, *doubly* Doppler Shifted by the motion of the rings and planet. The slant of the planetary lines is due to the apparent tangential velocity of the planet increasing towards the limb ($v = r\omega$). The opposite slant of the lines in the rings shows the decrease of orbital velocity of a particle with radius.

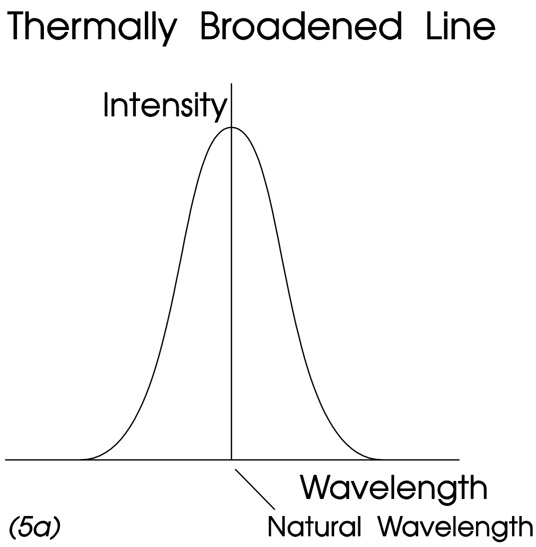
Can you show that $v \propto 1/\sqrt{r}$ for the rings? Plot out your results and compare them to a detailed spectrum of Saturn from a reference book.

Thermal Broadening

When an electron moves from one energy level to another in an atom, the energy difference can be related to the frequency of the photon (quantum) which is absorbed or emitted by the formula:

$E = hf$ where h is Planck's Constant = 6.6×10^{-34} J s.

This implies that the photon can only have one specific frequency for a given energy level jump. In fact, photons are observed to have a range of frequencies. The main contribution to this range is the Doppler Effect. In any gas above absolute zero, the molecules are in constant random motion. As they emit or absorb photons so their natural (rest) frequency will appear to increased or decreased as a result of the Doppler Effect.



(5a)

If the molecules had only velocities of $\pm v$ then using $\Delta\lambda/\lambda = -v/c$ the width of the natural line would be given by $\Delta\lambda/\lambda = 2v/c$. As molecules have a distribution of velocities so the most suitable measure is the root mean square (rms) velocity. The rms velocity (v_{rms}) of a monatomic gas molecule is related to its absolute temperature (T) by:

$\frac{3}{2}kT = \frac{1}{2}m\overline{v^2}$ where m is the mass of one molecule in kg.

$\therefore \overline{v^2} = \dfrac{3kT}{m}$ where $m = \dfrac{M_r}{1000} \Big/ N_A$

and $kN_A = R$

where N_A is Avogadro's Number,
M_r is the Relative Molecular Mass,
k is Boltzmann's Constant and
R is the Gas Constant.

Also $v_{rms} = \sqrt{\overline{v^2}}$

For atomic hydrogen at 273 K this gives v_{rms} = 2.61 km s^{-1}

Now the line width is given by:

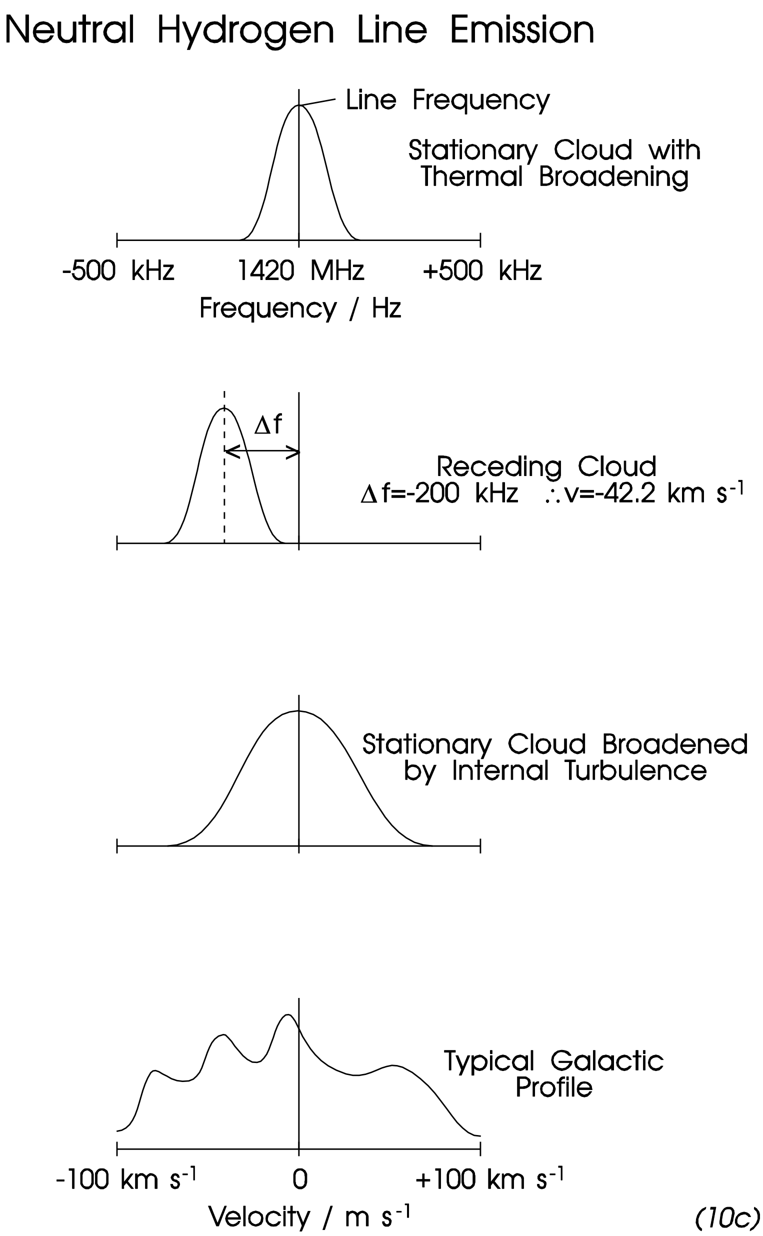
$$\Delta\lambda \propto v_{rms} \propto \sqrt{T}$$

Using the Lyman α line at 121.6 nm this gives $\Delta\lambda$ = 2.1 pm which is about 1/1000% change! However for the layers of gas above a hot star (> 10 000 K) the broadening is easily seen with a good spectroscope.

Hydrogen Line Emission

The hyperfine emission from neutral hydrogen (H I) at 1420 MHz (21 cm) is particularly important. It results from the chance formation of hydrogen atoms from free protons and electrons. Half the atoms are formed with the spins of the proton and electron parallel (c.f. two bar magnets held side by side with their north poles touching). The parallel atoms flip into their lower energy state with the emission of a precisely defined radio quantum. The lifetime of the parallel state is 1.1×10^7 years! However collisions in the interstellar medium reduce this to just a few hundred years. Fortunately there are such large numbers of atoms that this emission line is easily seen!!

The plane of our Galaxy is opaque at optical frequencies so it was the radio astronomers who were able to map the spiral arms using the 1420 MHz line. Each 5 kHz of Doppler Shift corresponds to a relative velocity of 1.056 km s^{-1}. Can you show this using the Doppler Effect formula? A typical series of spectra is shown in the following graphs.



(10c)

Objects Observed by Radar Echoes

This method is useful in finding the rotational period of cloud-covered planets such as Venus. Jupiter has a gas and liquid surface and is rather too distant to use Earth-based radars - the echoes would be too faint. *Can you see why the power of the echo falls with the fourth power of the distance?* Recently radar observations of Venus by the orbiting probe, Magellan, have been a great success, mapping surface details down to a few hundred metres in scale.

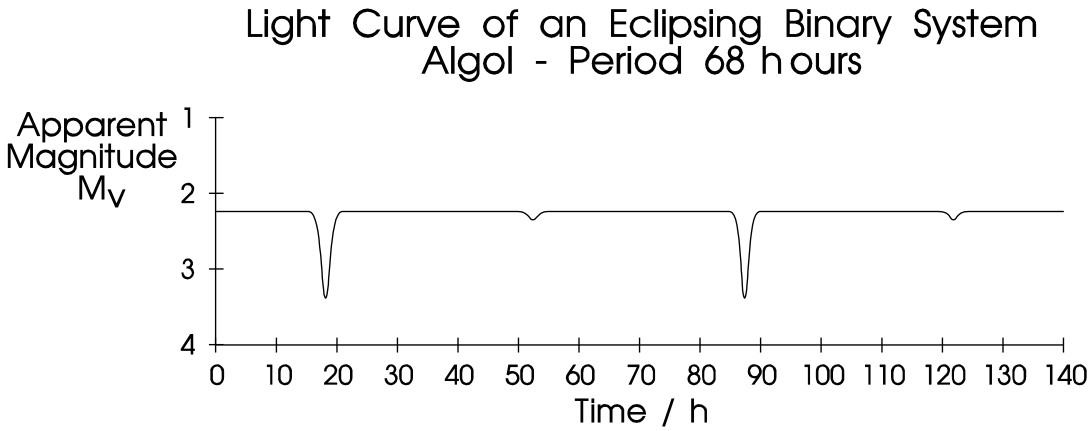
Double Doppler Effect occurs in the echoes. The time of flight of the pulse to the object and back gives a very accurate measurement of target to observer distance. The Doppler Shift of the signal gives the relative velocity.

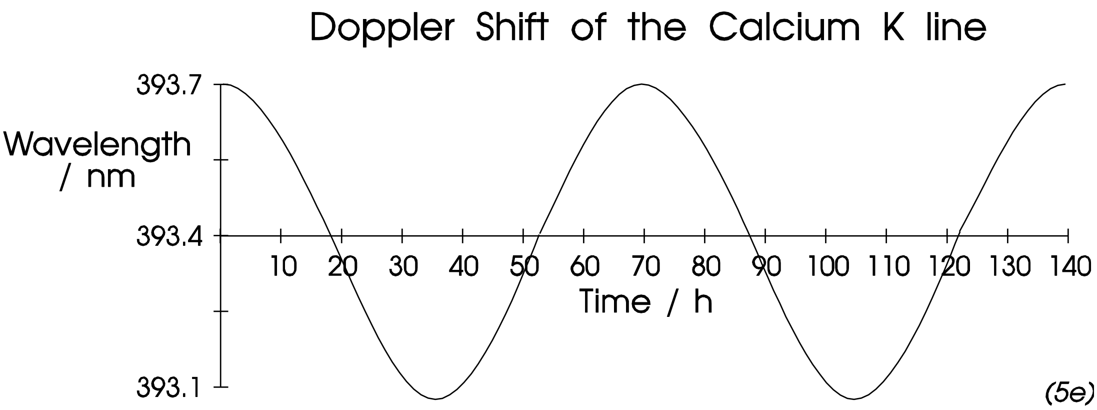
Radar implies radio waves but light may also be employed. Using a cat's-eye reflector left on the Moon by NASA, pulsed lasers on Earth can determine the Earth-Moon separation to within a few centimetres. This has improved the understanding of the tidal forces in the Earth-Moon system. These forces are causing the Moon to slowly recede from us.

N.B. In the previous sections it is assumed that the source, reflector and observer are in a straight line. If this is not so, then components of velocities must be used.

Eclipsing Binary Systems

Algol comprises a bright star and a faint star orbiting around their common centre of mass once every 68 hours. When the fainter star covers the brighter, the magnitude falls by 1.2 units. This effect is only observed because the Earth happens to lie in the plane of the orbits of the pair of stars. When an eclipse occurs there is no component of orbital velocity in our direction (all perpendicular) so the Doppler Shift of a spectral line from the system passes through zero once every 68/2 hours.





(5e)

Use the Doppler formula with data from the second graph to calculate the mean velocity of the stars in the Algol system? Could you run this fast to catch a bus?

Substitute this orbital velocity and the period derived from the first graph into the formula $T = 2\pi r/v$ *to calculate the mean distance (r) of the stars from their centre of mass. How does this distance compare to the distance between the Earth and the Sun?*

End of Extension Material

Cosmology

Hubble's Law

When Hubble first measured the red shifts of the galaxies he assumed that he was observing Doppler Shift:

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$ where $\Delta\lambda$ is the change in wavelength,

λ is the original wavelength in m,
v is the relative velocity in $\mathrm{m\,s^{-1}}$ and
c is the speed of light in $\mathrm{m\,s^{-1}}$.

The relative velocity v is taken as positive when the source and observer are approaching each other. The change in wavelength $\Delta\lambda$ will then be negative, implying an decrease in wavelength – a blue shift.

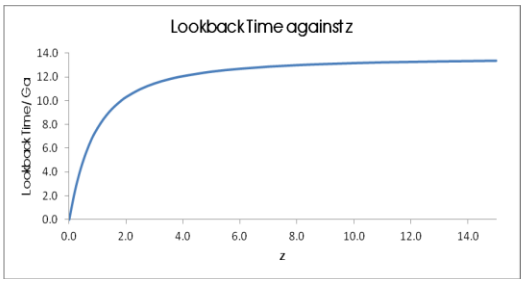
Red Shift

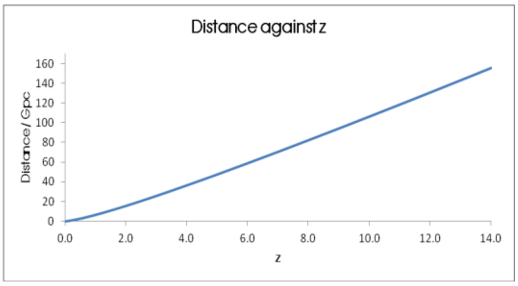
However it soon became apparent that the effect was caused by the expansion of the Universe. The same formula applies to both if the velocity is a small fraction of the speed of light.

When all but the most local (e.g. Andromeda – M31) galaxies are observed, the wavelengths of their emission and/or absorption lines are found to be longer than the corresponding ones as observed on the laboratory bench. It is generally accepted that this shows that the universe is expanding. The red shift is defined as:

$$\frac{\Delta\lambda}{\lambda} = z \quad \text{and} \quad z >= 0 \qquad \text{(Equation A)}$$

By taking spectra of the most distant galaxies it has recently proved possible to identify specimens with redshift $z = 8$. The light started on its journey when the Universe was only 700 million years old. It has been travelling for 13 billion years. No object has been found with significantly greater z and many galaxies have much smaller redshifts.





The Doppler formula may be expressed in terms of frequency:

Since $\lambda = \frac{c}{f}$ then differentiating $\frac{d\lambda}{df} = -\frac{c}{f^2}$

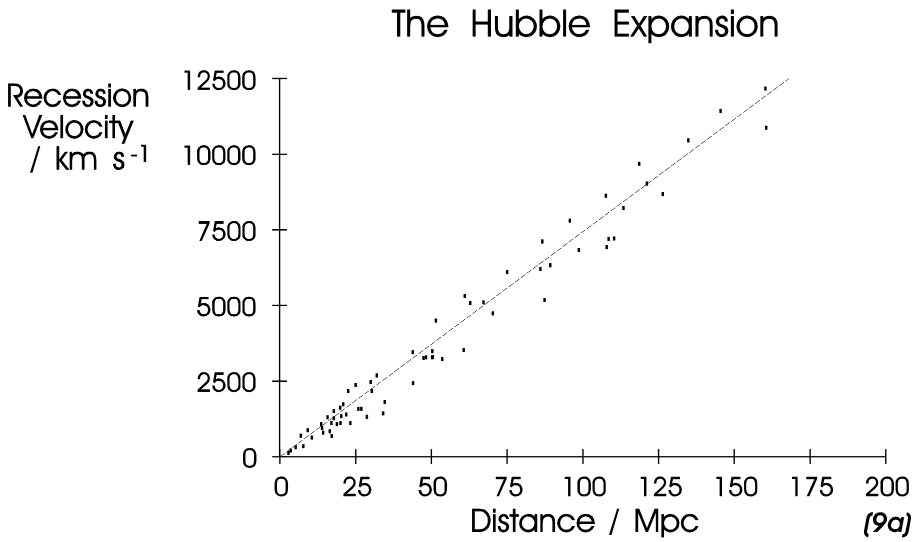
Substituting in equation A gives $\frac{\Delta f}{f} = z$

The minus sign can be ignored if it is noted that receding objects have a reduced apparent frequency.

In the real world, IR, visible, UV and X-ray astronomers use wavelength whereas radio astronomers mainly use frequency in their calculations!!

Recession of the Galaxies

The spectral lines of distant galaxies all show some red shift. If it is assumed that the redshifts are due to the velocities of the galaxies (a matter that has caused some controversy) then a link can be found between the distance of the galaxies and their recessional velocity.

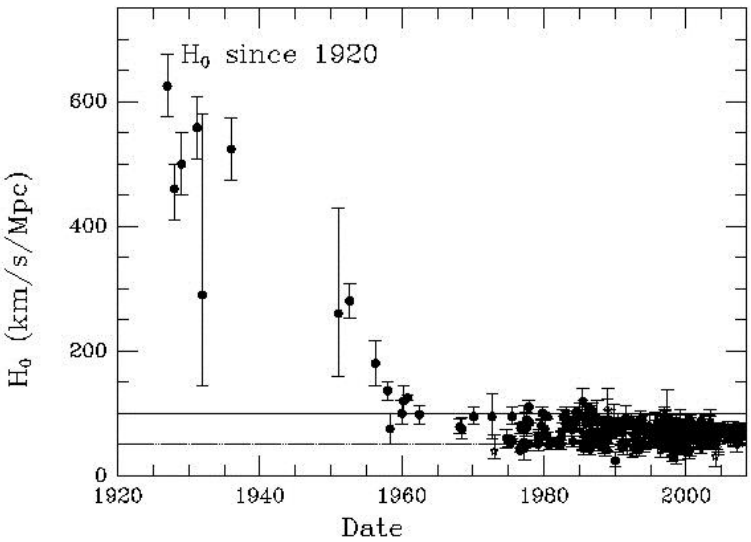


(9a)

Initially Hubble was only able to plot a trend line from the 46 galaxies he studied. He obtained a value for the gradient, now known as the Hubble Constant (H) of 500 km s^{-1} Mpc^{-1}. This was much higher than the currently accepted value due to errors in his distance calibrations.

When astronomy research started again after the Second World War, the value was revised to between 50 and 100 km s^{-1} Mpc^{-1}.

This graph, provided by the Harvard Centre for Astrophysics, shows the change in the accepted value of the Hubble Constant (H_0) since 1920:

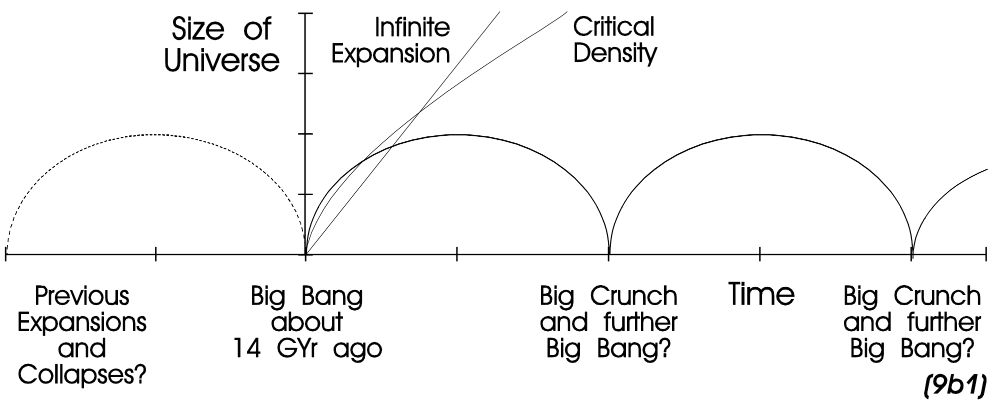


The most recent campaign using the Hubble Telescope to observe Cepheid Variable stars has yielded a value for the Hubble Constant (H) of $74.2 \pm 3.6\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$. The AQA Specification gives the value as $65\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ which should be used when answering AQA examination questions.

At the moment we can only measure the distance to Cepheid Variable stars out to about 30 Mpc. Beyond that we have to use Type 1a supernovae. These can be seen out to distances of perhaps 1000 Mpc but are so faint that they illuminate just one pixel on the CCD! Observations continue to secure more data.

If the redshift of a newly observed galaxy is used to calculate its velocity, then its distance may be read off the graph. At some great distance the recession velocity is $3 \times 10^8\,\text{m}\,\text{s}^{-1}$ – the velocity of light! *Can you calculate the distance to the edge of the observable Universe? Use this to calculate an estimate of the age of the Universe?*

The Fate of the Universe



(9b1)

When the Hubble graph is extended out to the most distant galaxies ($z > 4$) the data are weak so that it is not certain that the straight line law holds. If the shape of the line/curve could be found with certainty it would indicate the fate of the Universe. If the most distant galaxies are found to be moving away rapidly then the universe will expand forever. If they are moving too slowly then gravity will pull them back to the Big Crunch. If the velocities are just right for the density of matter, then the Universe will come to rest at infinite size.

As the data improves it will be possible to plot a more secure Hubble graph. From the shape it will be possible to link the expansion of the galaxies with the amount of matter in the Universe. At present the amount of mass estimated from visible matter falls short by between 10 and 100 times of that needed to close the Universe and so ensure a Big Crunch. Searches are continuing for black holes, brown dwarfs and molecular hydrogen clouds in particular, all of which are difficult to detect and are thought to hold a great deal of matter.

Cosmology

Quasars

First detected as very small, relatively bright radio sources (originally called radio stars) that had no obvious counterparts on the optical sky survey plates. In 1963 the Moon was due to pass in front of one of them (3C273). As the orbit of the Moon was accurately known, a precise position of the Quasar was found by recording the times of disappearance and reappearance of its radio signal. When the position was studied in visible light, a faint star with a few emission lines in its spectrum as found. This itself was a puzzle as most stars show absorption lines! The lines were not easily identified but by careful measurement it became apparent that they were from neutral hydrogen and ionised oxygen but with a significant redshift - not expected from a star!

The source was the now well-known quasar 3C273 (Object 273 in the third Cambridge radio survey). Its optical image consists of a 14th magnitude 'star' with a faint narrow jet about 10 arc seconds long. In the radio image, one part centres on the end of the jet and the other on the 'star'. The elongated image of the jet is thought to be caused by relativistic electrons spiralling in the local magnetic field and so emitting bright beams of synchrotron radiation.

When the distance of 3C273 was obtained from its red-shift ($z = 0.16$) it was found to be 1.5×10^9 light years away. Definitely outside our Galaxy!! However if it was to be a star at that distance then its luminosity would be far too large to be realistic (10^{39} W); perhaps implying 1000 times the power of our Galaxy being generated in a region the size of the Solar System!!

Try the following calculation. Take the absolute magnitude of the Sun to be 4.8 and calculate its magnitude if it was 1.5×10^9 light years away. You will need to change the distance to parsecs. Would you be able to see it with a large telescope? Compare you answer with the magnitude given for 3C273 in the previous paragraph. This should give an idea of how very bright quasars are!

From these arguments it is generally accepted that the red-shift is real and that the energy comes from a galactic nucleus at a considerable distance and *not* from a nearby starlike object.

However there was considerable controversy when quasars were first discovered. Perhaps their spectra were reddened by scattering of light as it passed through interstellar dust clouds? Perhaps the red shift was caused by the photons having to escape from an intense gravitational field? When you look at the optical spectrum of a quasar you realise how difficult it would have been to obtain anything useful from it, particularly when only a few examples where known. Now there is little argument. Many examples are known and very detailed and beautiful radio maps have been made.

It is quite likely that many galaxies pass through an energetic, quasar phase as they evolve or are disturbed by another near-by galaxy. It is now thought that there is a super-massive black hole at the centre of our Galaxy! The Black Hole swallows large quantities of matter during the energetic periods that may last 50 million years before the centre of the galaxy is cleared of material. *Is the Earth likely to be swallowed?* Seyfert galaxies and BL Lac objects seem to be less extreme forms of quasars.

Only by using VLBI (Very Long Baseline Interferometry) with radio telescopes on two continents has it become possible to put a lower limit on the size of the radio quasars and to find their positions accurately enough to link them to their optical images.

It is worth noting that 90% of the Quasars are radio quiet! The advent of automatic plate measuring machines has made the discovery of Quasars easier. The systems look for particular characteristics in the spectrum of the light to obtain a fair sample of the sources.

Detection of Exoplanets

Difficulties of Observation

If you look into the night sky you can easily see Venus, Mars, Jupiter and Saturn. However, trying to see Mercury is difficult. It orbits close to the Sun and so is lost in the bright blue light of our sky. Mercury is visible just before the Sunrise or just after Sunset if you have a clear horizon.

Now imagine looking back at the Sun from the nearest stars, 4 light-years away. The light from the Sun is so much brighter than the reflected light from any of the planets that you just can't see them. With a very high quality telescope in space, you could probably just see Jupiter and Saturn.

As the stars in our quadrant of the Galaxy extend out to perhaps 30 000 light-years, we need other ways to locate exoplanets.

Detection by Doppler Shift

The Moon orbits the Earth and to a first approximation you might think that the Earth did not move. However the Moon and Earth both orbit their common Centre of Mass (Barycentre). This is a point below the surface of the Earth (1 710 km down) on a line joining centres of the Moon and the Earth. So the Moon describes a large circle and the centre of the Earth a small one.

Now a planet orbiting its parent star will behave in the same way. As any planet is likely to be much less massive than its star the effect on the star will be much smaller.

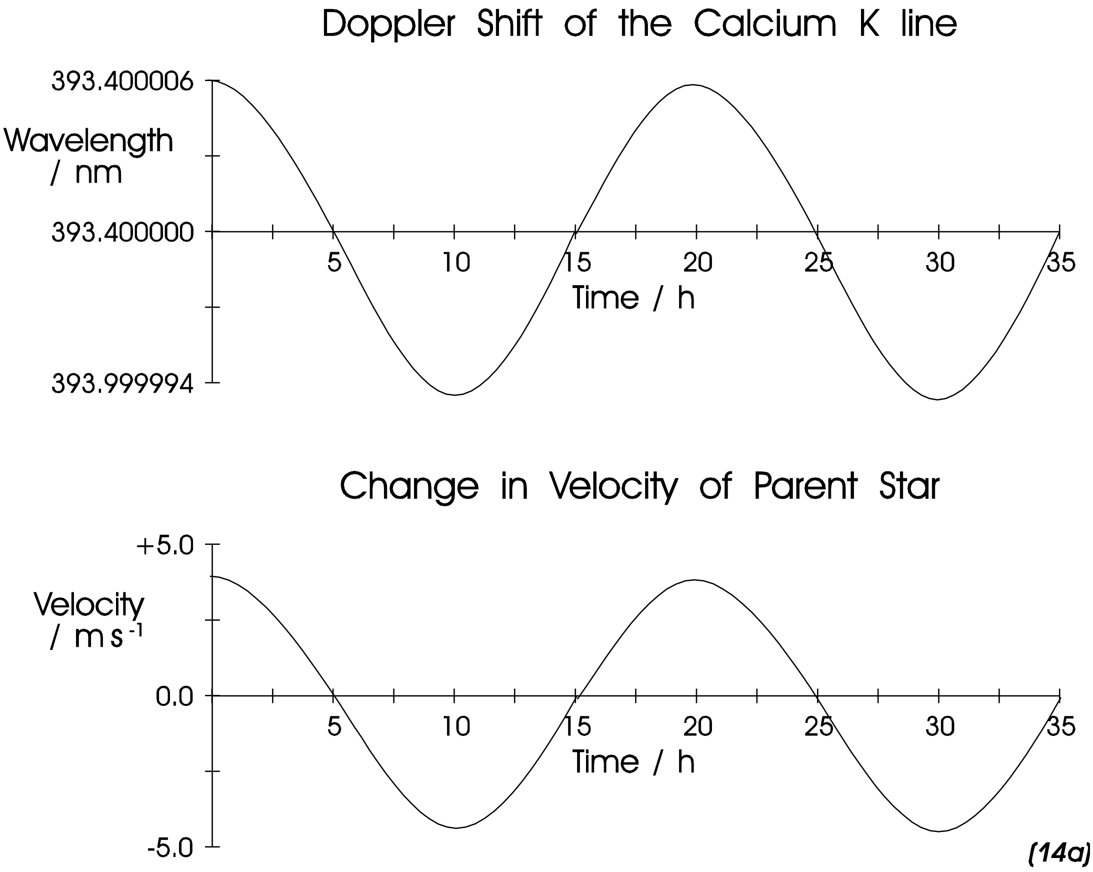
However, using a telescope that directs its light onto a diffraction grating it is possible to record the spectrum of the star. If the star has a planet then the surface of the star will slowly approach and recede from us. By observing the spectrum over many months it is possible to observe this effect in many stars.

The effect has been known for over 100 years and is used to find binary stars that are too close to each other or too far away from us to separate with a telescope. In the section on the Doppler Effect you can see the Calcium K line from the binary star Algol showing a shift in wavelength. The graph shows a shift from 393.1 nm to 393.7 nm and back in 69 hours. This is a velocity change of $4.6 \times 10^5 \text{ m s}^{-1}$ and is easy to observe.

Now consider a planet like Jupiter orbiting a star like our Sun. The effect is very much smaller. The velocity change in the surface of the Sun is about 12 m s^{-1}. This gives a change in wavelength of about 1.6×10^{-5} nm over a period of 12 years. Although very small, this is detectable with a precision instrument (e.g. HARPS at ESO). The smallest change in velocity detectable at the moment is $0.5\,\mathrm{m\,s^{-1}}$. This means that the Earth, with a change of about $0.1\,\mathrm{m\,s^{-1}}$, would not be detectable.

All the preceding discussion has assumed that we are in the plane of the orbit of the exoplanet. If we are at an angle to the plane, the effects will be diminished. If we are perpendicular to it, we will see nothing.

The graphs below show the change in wavelength of the Calcium K line and the corresponding change in velocity of the parent star for exoplanet Kepler-10b – the first unambiguously rocky exoplanet. As you can see from its period of 20 hours, it orbits very close to its star and so would be far too hot to sustain life.



Doppler effect measurement recorded using HIRES instrument at W.M. Keck Observatory showing the wobbling of the star due to Kepler-10b.
(Betalha et al. 2010, Astrophysical Journal)

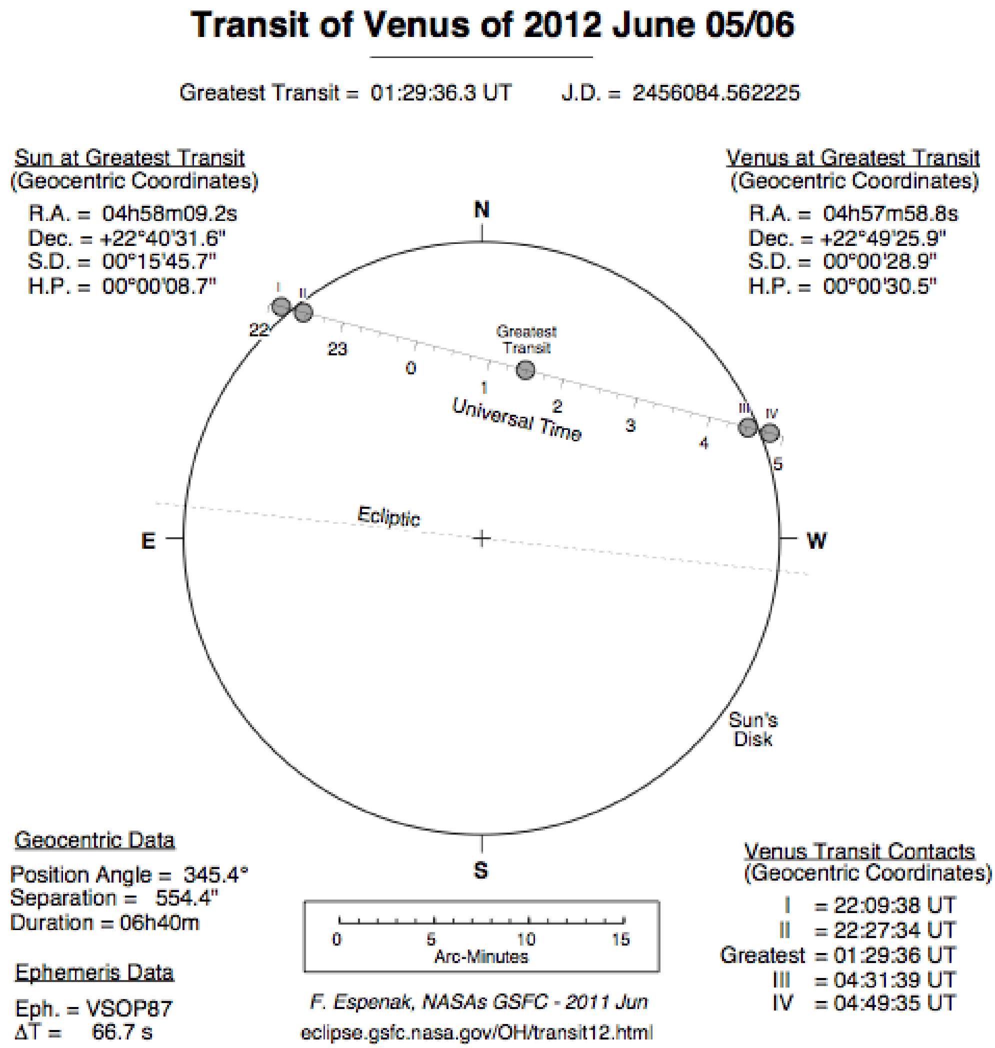
Detection by Transit Observation

A few years ago we were treated to the sight of a black dot passing slowly across the face of the Sun. It was a Transit of the planet Venus and it took about 6 hours in all. The next transit will be on the 10th December 2117. So from this we can see that transits are rare and don't last long!!

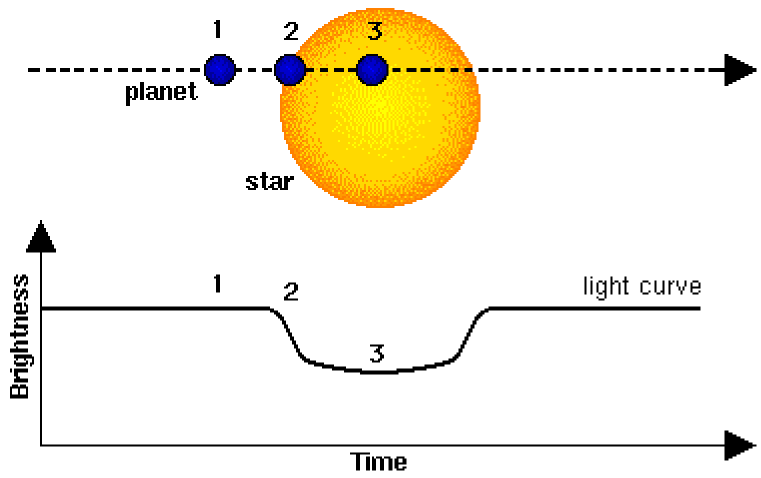
However it has proved possible to discover exoplanets using transits. Several tens have been observed from the ground and several hundreds have been observed by the Kepler spacecraft.

For a planet like Venus, the drop in brightness is about 0.1% so is not too difficult to measure if all other variables can be controlled. The longest transits will occur when we are in the plane of the exoplanet. As you can see from the diagram of the Venus transit, it would be very easy for the planet to pass above or below its star and so we would see nothing. For a planet orbiting a Sun-sized star at the distance of the Earth, the probability of a random alignment producing a transit is about 0.5%.

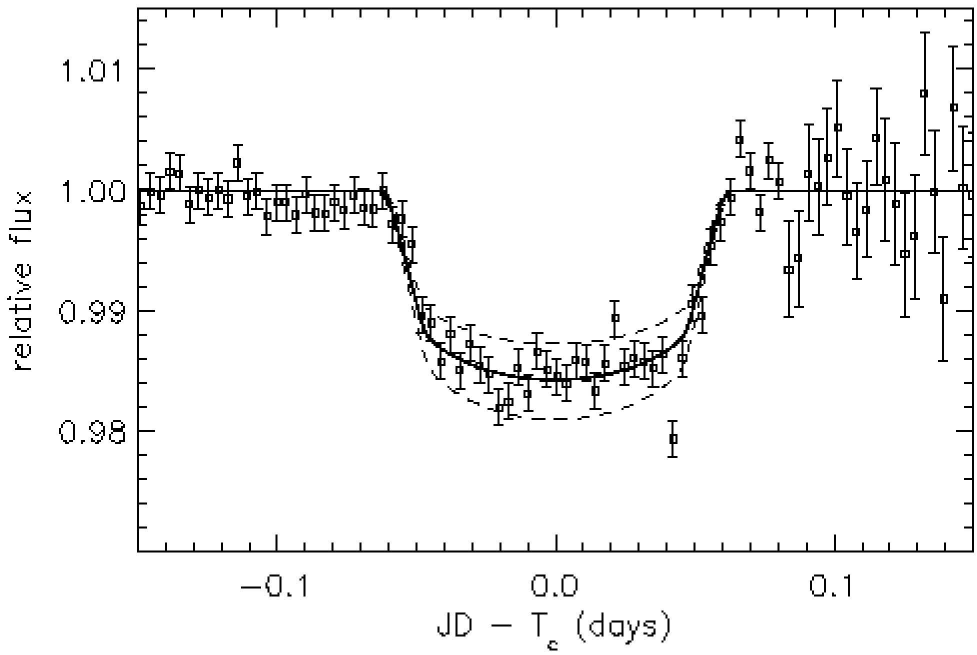
FIGURE 2



The diagram below shows the light curve from a planet moving in front of its star.



In 1999 astronomers led by David Charbonneau and Gregory W. Henry detected the first known transiting extrasolar planet – HD 209458 b. The star is dimmed by about 2% every 3.5 days.



Superposed light curves of star HD 209458 showing transits occurring on 9 and 16 September, 1999.

For an up-to-date list of exoplanets visit http://exoplanets.org/table

Books for Teachers

The Internet and Wikipedia have changed information gathering forever! However, having one or two reference books to hand never hurts:

Universe William J Kaufmann III & Roger A Freedman, Freeman. A massively comprehensive American college text. Excellent value for the A-level teacher.

A Journey through the Universe: Gresham Lectures on Astronomy
Ian Morison Cambridge University Press. £22.50 978-1107073463
A detailed and thorough presentation of the key topics in Astronomy and Astrophysics. An excellent refresher for a teacher embarking on the subject. Can easily be dipped-into.

A Concise Dictionary of Astronomy Jacqueline Mitton, OUP. (Amazon Second Hand) 0198539673 An excellent reference work for those difficult questions that pupils ask at awkward times!

A Dictionary of Astronomy Ian Ridpath, OUP. £12 0199609055

Colours of the Stars David Malin and Paul Murdin CUP (Amazon, second hand)

If you need to go observing:

Norton's Star Atlas Ian Ridpath (ed.), Longman. £13 0131451642. A most detailed set of maps and reference material.

Philip's Planisphere: Northern 51.5 Degrees. A disc map of the sky. Very useful for initial orientation.

Have a look at www.aae.org.uk for Amazon Purchasing Options.

For up-to-date information:

Astronomy Now - popular and well produced UK monthly newsstand magazine. Subscriptions £33. Suitable for all secondary schools.

If you want some more detailed information then the books by Chris Kitchen are suitable for any physicist who wants to brush up on topics in astronomy, spectroscopy and optics. The book by Michael Rowan-Robinson is the most comprehensive discussion of the distance scale that I have ever seen, as well as containing good background material on stellar evolution and cosmology. The book by Gareth Wynn-Williams is an excellent review of gas and dust clouds. It is easy to read and covers the material to a good depth of detail. The book by Sharov is an excellent biography of Hubble and ends with an excellent summary of the problems which face cosmologists. The book by Smoot details the search for the cosmic microwave background from early balloon flights to the success of COBE and is good read. The Physics of Startrek will improve your street cred!!

Stars, Nebulae and the Interstellar Medium
Chris Kitchen Adam Hilger

Astrophysical Techniques
Chris Kitchen IOP Publishing

Optical Astronomical Spectroscopy
Chris Kitchen IOP Publishing

The Cosmological Distance Ladder
Michael Rowan-Robinson Freeman

The Fullness of Space
Gareth Wynn-Williams CUP

Edwin Hubble the discoverer of the Big Bang Universe
Alexander S Sharov CUP

Wrinkles in Time
George Smoot Little, Brown & Co

The Physics of Startrek
Lawrence M Krauss Harper Collins

Have a look at www.aae.org.uk for Amazon Purchasing Options.

Resources

Centres

Armagh Planetarium and Observatory www.armaghplanet.com
College Hill, Armagh, Northern Ireland, BT61 9DB, 01861 523689

British Association of Planetaria www.planetarium.org.uk
Links to Fixed and Mobile (Inflatable) Planetaria

Jodrell Bank Science Centre and Arboretum www.jodrellbank.net
Macclesfield, Cheshire, SK11 9DL, 014775 71339

Liverpool Museums Planetarium www.liverpoolmuseums.org.uk

London Planetarium **No longer presenting astronomy shows!**

National Space Centre www.spacecentre.co.uk
Exploration Drive, Leicester, LE4 5NS

Norman Lockyer Observatory www.normanlockyer.org
Sidmouth, Devon

Royal Observatory Edinburgh Visitor Centre www.roe.ac.uk/vc
Blackford Hill, Edinburgh, EH9 3HJ, 0131 668 8405

The Royal Observatory Greenwich www.rmg.co.uk
Museum and Planetarium

Techniquest www.techniquest.org
Stuart Street, Cardiff, CF1 6BW, 01222 475475

University of Central Lancashire, Alston Hall Observatory
www.alston-observatory.uclan.ac.uk/

University of London Observatory www.ulo.ucl.ac.uk
Mill Hill Park, London, NW7 2QS, 020 8959 0421

Wynyard Woodland Park Observatory and Planetarium,
Stockton-on-Tees www.wynyard-planetarium.net

Magazines and Leaflets

Astronomy Now – popular and well produced UK monthly newsstand magazine with features such as A to Z of Astronomy, Sky Diary and current review articles. Subscriptions £33 information from Pole Star Publications Limited. Suitable for all secondary schools. www.astronomynow.com

Popular Astronomy – covers Astronomy and Spaceflight produced quarterly for members, who are often beginners in the subject. Articles cover observing, current space-flight events and other topical issues. Suitable for all secondary schools. Subscription £18 per year. The Society for Popular Astronomy www.popastro.com

Sky and Telescope – largest monthly magazine on Astronomy and with articles by eminent writers for the more advanced student. Would suit 4th year secondary upwards. Sky Publishing Corporation. $60 per year. www.skyandtelescope.com

Science and Technology Facilities Council – www.stfc.ac.uk has a large catalogue of resources including pamphlets and resource lists.

Catalogues

Astronomical Society of the Pacific, www.astrosociety.org

British Astronomical Association, http://britastro.org/baa
Burlington House, Piccadilly, London, W1V 9AG. 020 7743 4145.

Telescope House, www.telescopehouse.com

Earth & Sky, www.earthandsky.co.uk West Barsham Road, East Barsham, North Norfolk, NR21 0AR, 01328 820083. Extensive range of astronomy books by mail order.

Sky and Telescope, www.skyandtelescope.com

Books

The Universe at your Fingertips ed. A Fraknoi. A massive compilation of teaching material suitable for teachers to dip into. Middle school and above. Available from The Astronomy Society of the Pacific web site shop. DVD $30 + p&p.

Astronomy (Eyewitness Science Series) K Lippincott, Dorling Kindersley £10. Beautifully illustrated and rich in detail.

How the Universe Works (Eyewitness Science Series) Heather Couper & Nigel Henbest, Dorling Kindersley. Amazon Second Hand). 0751300802. Well illustrated and lots of things to do.

Guide to the Night Sky Patrick Moore, Philip's. £4 0540063150. Guided tour of the naked eye stars.

Space Encyclopedia Heather Couper and Nigel Henbest, Dorling Kindersley. 0751354139 An excellent modern encyclopedia covering theoretical and practical topics in the usual DK style. Apart from the odd missing equation, this work reaches A-level standard. A CD-ROM of Space and the Universe is included. (Amazon, second hand)

Have a look at www.aae.org.uk for Amazon Purchasing Options.

Software

Stellarium – Planetarium Software www.stellarium.org Free and excellent quality.

Redshift 7 – Planetarium Software www.redshift-live.com
CD-ROM Excellent value! Has images and texts easily accessible.

Equipment

Project Star Spectrometer (PS-14/Plastic) is available at $42 + p&p. Search Amazon on line. Supplier is Fisher Scientific.

Philip's Planisphere: Northern 51.5 Degrees. A disc map of the sky. Very useful for initial orientation. Search Amazon on line.

Astronomy on the Internet

Association for Astronomy Education	www.aae.org.uk
Astronomy Now Magazine	www.astronomynow.com
Cambridge Astronomy	www.ast.cam.ac.uk
Campaign for Dark Skies	www.britastro.org/dark-skies
European Association for Astronomy Education	www.eaae-astronomy.org
European Southern Observatory	www.eso.org
European Space Agency	www.esa.int
Hands On Universe Project	www.handsonuniverse.org
Hands On Universe Project in the UK	www.uk.euhou.net
Heavens Above	www.heavens-above.com
Hubble Space Telescope	http://hubblesite.org
Imperial College Impact Crater Site	http://impact.ese.ic.ac.uk/ImpactEffects
International Space Station	www,nasa.gov
NASA	www.nasa.gov
NASA Science	http://nasascience.nasa.gov
Royal Astronomical Society – Education Committee	www.ras.org.uk
Royal Observatory Edinburgh	www.roe.ac.uk
Search for Extra-Terrestrial Intelligence (SETI)	www.seti-inst.edu
Science and Technology Facilities Council	www.stfc.ac.uk
Space Calendar	www2.jpl.nasa.gov/calendar
Welcome to the Planets	http://pds.nasa.gov/planets

Useful Astronomical Data

Absolute Zero	–273.15 °C
Avogadro's Number	6.02×10^{23} mol^{-1}
Boltzmann's Constant	1.38×10^{-23} J K^{-1}
Charge of Electron	-1.60×10^{-19} C
Gravitational Constant	6.67×10^{-11} N m^2 kg^{-2}
Mass of Electron	9.11×10^{-31} kg
Mass of Proton	1.67×10^{-27} kg
Molar Gas Constant	8.31 J mol^{-1} K^{-1}
Molar Volume at stp	2.24×10^{-2} m^3 mol^{-1}
Permeability of Vacuum	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity of Vacuum	8.85×10^{-12} F m^{-1}
Planck's Constant	6.63×10^{-34} J s
Stefan's Constant	5.67×10^{-8} W m^{-2} K^{-4}
Velocity of Light	3.00×10^{8} m s^{-1}
Wien's Law Constant	2.90×10^{-3} m K
Sun	
Equatorial Radius	6.96×10^{8} m
Mass	2.00×10^{30} kg
Mean Density	1.42×10^{3} kg m^{-3}
Sidereal Rotational Period (Latitude 17°)	25.4 days
Luminosity	3.90×10^{26} W
Earth	
Equatorial Radius	6.37×10^{6} m
Mass	5.97×10^{24} kg
Mean Density	5.52×10^{3} kg m^{-3}
g approximately	9.81 m s^{-2}
Mean Distance to Sun	1.495×10^{11} m
Solar Constant	1.37×10^{3} W m^{-2}
Mass of Atmosphere about	$1 \times 10^{-6} \times$ Mass of Earth
Moon	
Equatorial Radius	1.74×10^{6} m
Mass	7.33×10^{22} kg
g approximately	1.62 m s^{-2}
Mean Distance to Earth	3.84×10^{8} m

(continued overleaf)

Mercury	
Equatorial Radius	2.44×10^6 m
Mass	3.30×10^{23} kg
g approximately	3.78 m s^{-2}
Mean Distance to Sun	5.791×10^{10} m
Mean Density	5.43×10^3 kg m^{-3}
Length of Day	58.65 days
Length of Year	87.97 days
Venus	
Equatorial Radius	6.05×10^6 m
Mass	4.87×10^{24} kg
g approximately	8.60 m s^{-2}
Mean Distance to Sun	1.082×10^{11} m
Mean Density	5.25×10^3 kg m^{-3}
Length of Day	243.0 days
Length of Year	224.7 days
Jupiter	
Equatorial Radius	7.15×10^7 m
Mass	1.90×10^{27} kg
g approximately	22.9 m s^{-2}
Mean Distance to Sun	7.783×10^{11} m
Mean Density	1.33×10^3 kg m^{-3}
Length of Day	9.84 hours
Saturn	
Equatorial Radius	6.03×10^7 m
Mass	5.69×10^{26} kg
g approximately	9.05 m s^{-2}
Mean Distance to Sun	1.427×10^{12} m
Mean Density	0.69×10^3 kg m^{-3}
Length of Day	10.2 hours
Light Travel Times	
Earth to Sun	8.31 minutes
Sun to Pluto	5.46 hours
Sun to Nearest Star	4.2 years
Across our Galaxy	1×10^5 years
To Andromeda (M31)	2.2×10^6 years
To the Edge of the Observable Universe	1.37×10^{10} years